ASSET EXEMPTION IN BANKRUPTCY, ACCESS TO AND COST OF CREDIT*

Pasqualina Arca† Gianfranco Atzeni‡ Luca G. Deidda§

Abstract

Under the US personal bankruptcy law, exempt assets are not liquidated following bankruptcy. Entrepreneurs can undo such a protection by posting collateral. We provide a complete characterization of the interplay between asset exemption from liquidation upon default and adverse selection in a competitive credit market. Severe adverse selection induces separation, with safer entrepreneurs choosing loan contracts characterized by high collateral requirements, lower cost of credit and credit rationing for wealth-constraints applicants. Irrespective of adverse selection, poor safe entrepreneurs pool as they face too much rationing, otherwise. Higher exemption makes collateral more informative. Evidence from the SSBF survey supports our theory.

Jel Codes: D82, E51, G33, K35

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† Università di Sassari.

‡ Università di Sassari and CRENoS

§ Corresponding author. Università di Sassari and CRENoS. Address: Dipartimento di Scienze Economiche e Aziendali, Università di Sassari, Via Muroni 25, 07100, Sassari, Italy. Tel. +39.3332514329, fax +39.079.213002. E-Mail: deidda@uniss.it.

1 Introduction

If an individual entrepreneur files for bankruptcy under Chapter 7 of the US personal bankruptcy law to repay creditors, the trustee appointed by the bankruptcy court would liquidate only non-exempt assets. [1] Nevertheless, "[...] a valid lien (i.e., a charge upon specific property to secure payment of a debt) that has not been voided (i.e., made unenforceable) in the bankruptcy case will remain after the bankruptcy case. Therefore, a secured creditor may enforce the lien to recover the property secured by the lien. [...]". [2] In other words, if the debt is secured by a valid charge upon specific assets (i.e., collateral), secured creditors can still enforce their rights and liquidate the assets. That is, entrepreneurs can undo the debtor protection by posting enough collateral, which has an opportunity cost. In the event of default, an entrepreneur would lose the assets posted as collateral, while she would keep at least the exempted ones had she not offered them as collateral. The opportunity cost of posting collateral increases with the level of exemption and crucially varies across entrepreneur's types. Specifically, it is lower for relatively safe entrepreneurs compared to risky ones, as the former exhibit a lower probability of default than the latter. Accordingly, the policy provision of asset exemption established by the bankruptcy law, implies that the decision to post collateral might play an informative role.

Studying the relationship between asset exemption from liquidation in the event of default and the effectiveness of collateral as a sorting device, we uncover a novel set of predictions about the effect of such provision of the US personal bankruptcy law under Chapter 7 on access to and cost of credit in competitive credit markets characterized by adverse selection. We analyze a competitive credit market populated by entrepreneurs and lenders. Entrepreneurs are heterogeneous in their chances of success and personal wealth and demand credit to finance their enterprise. Lenders cannot observe and verify entrepreneurs' riskiness and screen applicants by offering a menu of contracts that specify the probability of access to credit, the collateral requirements, and the credit cost. Our

[1] This remains true after the 2005 reform. According to the US Courts' official website, https://www.uscourts.gov/services-forms/bankruptcy/bankruptcy-basics/chapter-7-bankruptcy-basics, 99% of undismissed or unconverted cases receive a discharge, and the trustees liquidates only non-exempt assets. According to the same source, in many cases, "[..] Chapter 7 cases are zero assets cases [..]". This anecdotal evidence suggests that the insurance effect implied by asset exemption could indeed be quite significant.

[2] This quote is taken from the following webpage of the official US federal courts' website: http://www.uscourts.gov/FederalCourts/Bankruptcy/BankruptcyBasics/DischargeInBankruptcy.aspx. As a clarifying example of the fact that exemption does not protect assets voluntarily posted as collateral, read about the case of Minnesota: http://www.legalconsumer.com/bankruptcy/bankruptcy-law.php?ST=MN. "[..] The investors who take the least risk are paid first. For example, secured creditors take less risk because the credit that they extend is usually backed by collateral, such as a mortgage or other assets of the company. They know they will get paid first if the company declares bankruptcy'[..]', http://www.sec.gov/investor/pubs/bankrupt.htm, U.S. Securities and Exchange Commission.

key findings are as follows. With zero asset exemption, the equilibrium, which always exists and it is characterized by a unique outcome, involves pooling across risk-heterogeneous entrepreneurs at any level of entrepreneurial wealth. Posting collateral is uninformative. Differently, with positive asset exemption, collateral becomes an effective screening device since the opportunity cost of posting collateral becomes relatively lower for safer entrepreneurs than for riskier ones. Accordingly, if adverse selection is sufficiently severe, the equilibrium is characterized by separation for sufficiently high levels of entrepreneurial wealth, with safer entrepreneurs self-selecting into contracts characterized by higher collateral requirements, a lower cost of credit and possibly a lower probability of access to credit. Separation is associated with rationing for safer entrepreneurs who decide to separate and are not wealthy enough to meet the collateral requirements to be financed with a probability equal to one. Irrespective of adverse selection, the equilibrium is always characterized by pooling at sufficiently low levels of entrepreneurial wealth. This result is equivalent to that found by Martin (2009), who studies the relationship between entrepreneurial wealth and aggregate investment under adverse selection. That is, collateral becomes an ineffective screening tool when entrepreneurs are sufficiently poor. Therefore, also in our setup pooling emerges either because entrepreneurs are very poor or adverse selection is not sufficiently severe. Accordingly, if adverse selection is sufficiently severe, a non-monotonic relationship between the probability of access to credit and entrepreneurial wealth emerges. This is due to the fact that among safe entrepreneurs those who are very poor pool and are always financed, while those with intermediate levels of wealth separate and face a positive probability of rationing. This result is in line with Martin (2009), who finds a non monotonic relationship between entrepreneurial wealth and investment.

An increase in the level of asset exemption enhances the informative role of collateral. Conditional on posting collateral, the cost of credit is further reduced, and access to credit is enhanced for those entrepreneurs who self-select into contracts characterized by the possibility of rationing. Still, the effect of an increase in exemption on aggregate credit rationing is uncertain. The fact that as exemption increases, entrepreneurs who separate by posting collateral face a lower probability of being rationed, reduces credit rationing in the market. However, an increase in exemption might also trigger an increase in the mass of wealth-constrained safe entrepreneurs who decide to separate, and consequently face to a lower probability of accessing to credit, which pushes up credit rationing. The net result of these two contrasting effects is ambiguous and depends on the shape of the wealth distribution of safe entrepreneurs' population.

We test our model's key implications using US data from the Survey of Small Business Finances

(SSBF) and exploiting the cross-state variability in exemption levels. A broad empirical literature investigates the effects of asset exemption using this dataset, which provides a strong comparability motive to its use. Relevant to our paper Gropp, Scholz, and White (1997) find that exemption reduces access to credit. Berkowitz and White (2004) find that high homestead exemption results in a greater chance of being denied credit and in a higher cost of credit for small businesses. Berger, Cerqueiro, and Penas (2011) who use the same wave of the survey as we do and introduce an individual-specific measure of asset exemption based on comparing individual home equity and homestead exemption, find that exemption induces higher interest rates and lower access to credit.[3] These findings are also confirmed by our estimations. That is, a higher exemption increases both the probability of credit rationing and the cost of credit. More importantly, we contribute to the above empirical literature by using our theoretical model as an identification tool to test key novel predictions about the combined effects of exemption and the decision to post collateral on access to credit and the cost of credit. In line with our model, the data confirm that while higher exemption and the decision to post collateral are, individually, negatively associated with access to credit, firms posting collateral are less likely to be rationed the higher the exemption level is. A similar conclusion holds for the cost of credit. The standard result is confirmed that posting collateral causes a reduction in the cost of credit. Crucially, in line with our model, we find this effect of being stronger the higher the exemption level is. Our estimation provides evidence that borrowers who decide to post collateral face a lower cost of credit and lower access to credit. Such evidence is perfectly consistent with our approach based on private information about borrowers' types. Safe borrowers posting collateral, but not offering enough guarantees, are willing to accept a lower probability of accessing credit to inform lenders' about their type, gaining a lower cost of credit. Therefore, in line with Berger, Cerqueiro, and Penas (2011), our empirical findings offer support for the idea that the use of collateral reflects the presence of ex-ante asymmetric information. These results are consistent with the idea of collateral being a signal of quality, which confirms what Jimenez, Salas, and Saurina (2006) find for a sample of Spanish firms.

From a theoretical perspective, related to our paper, Manove, Padilla, and Pagano (2001) show that excessive creditor protection (i.e., too little asset exemption) might induce a lazy attitude among banks using their costly screening technology to assess borrowers. Complementary to that, our analysis highlights that a reduction in the degree of creditor protection in the form of asset exemption gives lenders the incentive to screen applicants using collateral. Krasa, Sharma, and

[3] Berkowitz and Li (2000), find an equivalent effect in the mortgage market concerning access to credit.

Villamil (2008) and Tamayo (2015) study the effect of creditor protection on the cost of credit and probability of bankruptcy in a costly state verification environment. In these models, creditor protection is measured by the percentage of assets that firms retain in the event of default. This variable is treated as exogenous, so that the concept of creditor protection in these models differs from that implied by the asset exemption under Chapter 7. As we show in our model, the effect of asset exemptions can be undone by the decision to post collateral, which implies that the percentage of assets that a firm retains in the event of bankruptcy is – even in the presence of exemptions – endogenously determined.

Our model setup borrows from Besanko and Thakor (1987) with four crucial differences. First, we characterize the equilibrium for any possible value of asset exemption. Differently, they consider an economy where, in the event of default, creditors can satisfy their right to borrowers' assets only up to collateral value. This corresponds to the special case of unlimited exemption in our setup. Second, in line with Hellwig (1987), we explicitly model competition as a three-stage game. At stage one, the uninformed party (lenders) offers the contracts; at stage two, entrepreneurs apply for credit and choose among the contracts on offer; at stage three, lenders can reject or accept any of the applications they receive. Third, introducing this third stage ensures the existence of a subgame perfect equilibrium in pure strategies for all parameter configurations. The equilibrium always exists and the equilibrium outcome is uniquely determined. [4] Accordingly, by modeling competition as a three-stage game, we are able to fully characterized the interplay between adverse selection, effectiveness of collateral as a sorting device in the presence of asset exemption and entrepreneurial wealth. Only if adverse selection is sufficiently severe and asset exemption is non-zero, collateral plays a role as screening device, which results in a separating equilibrium outcome for levels of entrepreneurial wealth sufficiently high. Otherwise, the equilibrium is characterized by pooling at all levels of entrepreneurial wealth, and collateral is uninformative. Fourth, in our model, the credit market is populated by potential borrowers heterogeneous in terms of wealth, enabling us to uncover the implications of exemption in terms of credit rationing at the aggregate level.

More broadly, our analysis contributes to the vast literature on bankruptcy law and entrepreneurial activity. Elul and Gottardi (2015) analyze the beneficial incentive effects that debt forgiveness (augmented by "forgetting" default) might have on entrepreneurial activity and welfare. Fan and White

[4] For any given level of entrepreneurial wealth, irrespective of whether the equilibrium implies separation or pooling, it always delivers the contract most preferred by safe entrepreneurs. See also Martin (2005, 2009).

(2003) show that if individuals are risk-averse, asset exemption should increase their willingness to become entrepreneurs. Indeed, they estimate that the probability of owning a business is 35% for households living in states with unlimited exemption rather than a low exemption. Akyol and Athreya (2011) analyze the effects of bankruptcy exemption on individuals' attitudes toward self-employment by examining the trade-off between the insurance and the cost of credit effects induced by higher exemption levels. Debtor protection and asset exemption have been recently investigated in relation to consumption smoothing (Pattison, 2020), households debt (Severino and Brown, 2017) and consumer access to credit (Romeo and Sandler, 2021). Pattison (2020) also provides an assessment of the overall welfare effects. Likewise, our analysis provides insights on the welfare effects of the asset exemption policy in a market with adverse selection. With regards to the distributional effects, exemption makes safer entrepreneurs better off while riskier types are worse off. However, exemption might result in inefficient underinvestment.

The paper is organised as follows. In sections 2 and 3, we present the model and its main results. In section 4, we discuss the equilibrium effects of exemption on cost of credit and credit rationing. In section 5, we develop the empirical analysis. Section 6 concludes the paper.

2 The model

We consider a competitive market populated by a large number, E, of entrepreneurs and a large number, L, of lenders. Both entrepreneurs and lenders are risk-neutral. Each entrepreneur is endowed with an investment opportunity of size one and an amount of wealth, $w \in [0, \overline{w}]$. Entrepreneurs have no financial resources so that they need to borrow to finance investments. Each investment opportunity delivers $R > 0$ with probability p_θ and 0 otherwise, where p_θ is conditional on the entrepreneur's type, $\theta = H, L$, with $p_H > p_L$. Accordingly, we use "type-H" and "safe" to refer to entrepreneurs with $\theta = H$, and "type-L" and "risky" to refer to entrepreneurs with $\theta = L$. We assume that $p_L R > 1 + r$, meaning that entrepreneurs are worth financing irrespective of their type. Nature assigns types so that an entrepreneur is of type-H with probability μ and of type-L with probability $1 - \mu$. Lenders are endowed with one unit of financial resources each and face an opportunity cost of capital, $r > 0$. With no loss of generality, we set $L/E > 1$, such that financial resources are abundant. Entrepreneurial wealth, w, is illiquid: one unit of entrepreneurial wealth is worth $\beta < 1$ to other agents. Ex-ante information about the wealth and type of individual entrepreneurs is private. However, ex-post, wealth is observable and verifiable. Entrepreneurs can credibly disclose their wealth at no cost, ex-ante.

2.1 *Fresh start opportunity* and the role of collateral

In the event of default, lenders can seize entrepreneurs' wealth. However, we assume that the bankruptcy law guarantees entrepreneurs a fresh start opportunity in the following sense. If an entrepreneur endowed with an amount of wealth w defaults, lenders can appropriate his wealth only up to the non-exempt value, $\max\{w - \eta, 0\}$. where $\eta \geq 0$ is the value exempted from liquidation and therefore not seizable by lenders. Nevertheless, consistent with US personal bankruptcy law, we further assume that the exemption does not apply to wealth posted as collateral. Accordingly, if an entrepreneur posts collateral $C \geq 0$, the wealth that lenders can appropriate is $\max\{w - \eta, C\}$.

2.2 Timing

The time line of events is as follows:

Stage 0 : Nature assigns entrepreneurial types;

Stage 1 : Lenders simultaneously offer credit contracts;

Stage 2 : Entrepreneurs decide whether to disclose information about their wealth and whether to demand credit and under which contract;

Stage 3 : Lenders decide whether to reject or approve each loan application they receive;

Stage 4 : Exchange, if any, takes place.

2.3 Lending contracts

We define a lending contract $\mathcal{C}$ as a quadruple, $\mathcal{C} = \{l, R^B, C, \pi\}$, where l is the size of the loan, R^B is the value of the loan at maturity, C is the amount of collateral, and π is the probability of having access to credit. However, since each entrepreneur has access to a unit investment project and has no financial resources, each entrepreneur either demands one unit of financial resources or nothing. Therefore, with no loss of generality, we impose $l = 1$, so that, $\mathcal{C} = \{1, R^B, C, \pi\}$. For simplicity, in the subsequent discussion, we drop the first element that refers to the unitary loan size, and we describe a lending contract as $\mathcal{C} = \{R^B, C, \pi\}$. Note that, since loans are of size 1, R^B also measures the gross lending rate. For each loan, the maximum level of non-exempt wealth that lenders are entitled to seize in the event of default is R^B/β, which corresponds to a liquidation value equal to the value of the loan, R^B. Then, taking the level of asset exemption, η, into account, the real guarantees implicitly offered by an entrepreneur endowed with wealth w if applying for the contract $\mathcal{C} = \{R^B, C, \pi\}$ is

$$G = \min\{\max\{w - \eta, C\}, \frac{R^B}{\beta}\}. \tag{1}$$

We note that G is (weakly) increasing in C and (weakly) decreasing in η.

2.4 Agents' strategies and payoffs

The expected payoff of a type-θ entrepreneur, with $\theta \in \{L, H\}$, who signs a contract, $\mathcal{C}$, is

$$u_\theta = \pi[p_\theta(R - R^B) - (1 - p_\theta)G] + w. \tag{2}$$

Correspondingly, the expected payoff of a lender who finances that entrepreneur is

$$v_\theta = p_\theta R^B + (1 - p_\theta)\beta G. \tag{3}$$

2.5 Collateral as a screening device: the role of asset exemption, η

Let $\mathcal{C}_1$ and $\mathcal{C}_2$ be two contracts with $\pi_1 = \pi_2 = 1$, $C_1 > C_2$ and $R_1^B < R_2^B$, such that the associated levels of guarantees, G_1 and G_2, satisfy $G_1 > G_2$. [5] It is immediate to verify that, given $p_L < p_H$,

$$p_L(R - R_1^B) - (1 - p_L)G_1 \geq p_L(R - R_2^B) - (1 - p_L)G_2 \tag{4}$$

implies

$$p_H(R - R_1^B) - (1 - p_H)G_1 > p_H(R - R_2^B) - (1 - p_H)G_2. \tag{5}$$

Whenever a risky entrepreneur prefers the contract characterized by a higher level of guarantees, a safe entrepreneur strictly prefers that contract. This sorting condition implies that type-H (type-L) entrepreneurs could self-select into contracts characterized by a level of guarantees that is comparatively high (low). Accordingly, collateral is a sorting device since guarantees are weakly increasing in collateral. As we shall see, its effectiveness depends upon the level of exemption, η. The intuition is as follows. Under no exemption, i.e., if $\eta = 0$, in the event of default, entrepreneurs' wealth is liquidated irrespective of whether they post any collateral. Hence, posting collateral does not provide any meaningful information. In the opposite extreme case of unlimited exemption,

[5] Other things equal, given equation (1), $C_1 > C_2 \Leftrightarrow G_1 > G_2$ for values of η sufficiently high and values of β sufficiently low.

i.e., if $\eta \to \infty$, entrepreneurs' wealth is liquidated in the event of default if and only if they post it as collateral. Accordingly, since the sorting condition applies so that type-L entrepreneurs dislike posting collateral more than type-H ones, the decision to post collateral becomes potentially informative about the type of the entrepreneur posting it.

3 Equilibrium analysis

We focus on subgame perfect equilibria. [6]

Definition 1. *An equilibrium is a strategy profile for lenders and entrepreneurs such that at each node of the game, players' strategies for the remainder of the game are the best replies given the strategies of the other players.*

We first characterize the separating and pooling equilibrium contracts for a given level of wealth. Then, we study equilibrium existence and characterization for any level of entrepreneurial wealth w. With no loss of generality, we focus on parameter configurations such that the following restrictions hold.

Assumption 1 (Entrepreneurs' participation and loan riskiness)**.**

1. *For $\theta \in \{L, H\}$,*

$$p_\theta R - (1+r)\left[p_\theta + \frac{1-p_\theta}{\beta}\right] > 0; \tag{6}$$

2. *The highest value of entrepreneurial wealth, $\overline{w}$, satisfies*

$$\overline{w} < \frac{1+r}{\beta} + \eta. \tag{7}$$

As we shall see, condition (6) ensures that, in any equilibrium, all entrepreneurs make strictly positive expected profits if financed so that they demand credit. Condition (7) implies that even for the wealthiest entrepreneurs, the liquid value of non-exempt individual wealth, $\beta \max\{\overline{w} - \eta, 0\}$, would be not enough to compensate lenders' opportunity cost. Accordingly, all loans are risky.

3.1 Separating contracts

For a given level of wealth, w, separation involves a menu of contracts, $\{\mathcal{C}_\theta = \{R^B_\theta, C_\theta, \pi_\theta\}; \theta = H, L\}$ where $\mathcal{C}_\theta$ is the contract offered to a borrower of type-θ, such that:

[6] Since the player who moves first has no private information, subgame perfect equilibria are equivalent to perfect Bayesian equilibria and sequential equilibria.

1. Borrowers' incentive compatibility constraints (ICC) are satisfied:

$$(ICC_H) \quad : \quad \pi_H[p_H(R - R_H^B) - (1 - p_H)G_H] \geq \pi_L[p_H(R - R_L^B) - (1 - p_H)G_L]; \tag{8}$$

$$(ICC_L) \quad : \quad \pi_L[p_L(R - R_L^B) - (1 - p_L)G_L] \geq \pi_H[p_L(R - R_H^B) - (1 - p_L)G_H]. \tag{9}$$

2. Borrowers' participation constraints are satisfied:

$$(PC_\theta): \qquad \pi_\theta[p_\theta(R - R_\theta^B) - (1 - p_H)G_\theta] \geq 0, \quad \theta = H, L. \tag{10}$$

$$\tag{11}$$

3. Lenders' participation constraints (PCs) conditional on the type of financed entrepreneur are satisfied:

$$p_\theta R_\theta^B + (1 - p_\theta)G_\theta\beta \geq (1 + r), \quad \theta = H, L. \tag{12}$$

4. Feasibility constraints are satisfied: $G_\theta \geq 0$, $G_\theta \leq w$, $\pi_\theta \in [0, 1]$, $\theta = H, L$,

where G_L and G_H are the levels of guarantees as given by equation (1), given the contract $\mathcal{C}_\theta$ designed for type-θ entrepreneurs, with $\theta = L, H$. The following result holds,

Lemma 1 (Equilibrium separating contracts: Characterization)**.** *In equilibrium, if separation occurs among entrepreneurs endowed with the level of wealth, w, the outcome is unique in terms of cost of credit, guarantees and access to credit. All entrepreneurs demand credit, and lenders offer a menu of lending contracts, $\{\mathcal{C}_\theta; \theta = H, L\}$, such that*

$$R_\theta^B = \frac{(1+r)}{p_\theta} - \frac{(1-p_\theta)\beta G_\theta}{p_\theta}, \tag{13}$$

$$C_H = \min\{\frac{(1+r)(p_H - p_L) + p_H(1-p_L)(1-\beta)G_L}{(1-p_L)p_H - p_L(1-p_H)\beta}, w\} \Rightarrow G_H = C_H, \tag{14}$$

$$C_L \in [0, w - \eta] \Rightarrow G_L = \max\{w - \eta, 0\} \tag{15}$$

$$\pi_L = 1, \tag{16}$$

$$\pi_H = \min\{\frac{p_L R - (1+r) - (1-p_L)(1-\beta)G_L]}{[p_L R - \frac{p_L}{p_H}(1+r) - (1-p_L)\left[1 - \frac{p_L}{p_H}\frac{1-p_H}{1-p_L}\beta\right] w}, 1\}, \tag{17}$$

$$u_\theta = p_\theta - (1+r) - (1-\beta)(1-p_\theta)G_\theta. \tag{18}$$

Proof. See appendix.

In equilibrium, if separation takes place, entrepreneurs of type-H separate from type-L ones by self-selecting into contracts characterized by a higher level of guarantees and a lower (or equal) probability of access to credit than those associated with contracts for type-L borrowers. Equation (17) implies that entrepreneurs endowed with wealth $w \geq \widehat{w}_1$, where

$$\widehat{w}_1 \equiv \begin{cases} \frac{(1+r)}{\beta} - \eta \frac{(1-p_L)(1-\beta)p_H}{(p_H-p_L)\beta} & \text{if } \eta \leq \overline{\eta} \equiv \frac{(1+r)(p_H-p_L)}{(1-p_L)p_H-(1-p_H)p_L\beta} \\ \frac{(1+r)(p_H-p_L)}{(1-p_L)p_H-(1-p_H)p_L\beta} & \text{if } \eta \geq \overline{\eta}, \end{cases} \tag{19}$$

are *rich* in the sense that they can afford the level of real guarantees necessary to self-select into contracts designed for type-H entrepreneurs and characterized by a probability of access to credit equal to one. In contrast, entrepreneurs with $w < \widehat{w}_1$ are *poor*, as they cannot afford such level of real guarantees. Accordingly, rich type-H entrepreneurs self-selecting themselves into separating contracts are financed with probability one, while poor type-H entrepreneurs face a positive probability of rationing. For rationed entrepreneurs, the marginal effect of an increase in wealth, w, on the probability of access to credit, π_H, is strictly positive.[7] Intuitively, lenders have two instruments to separate safe from risky borrowers: collateral and the probability of access to credit. Poor entrepreneurs post all their wealth as collateral and separate from risky borrowers by accepting a lower probability of access to credit. Finally, it is immediate to verify that, for any level of wealth, w, the level of guarantees associated with separating contracts designed for type-H entrepreneurs is strictly higher than that of the correspondent contracts for type-L entrepreneurs. Accordingly, given that type-H entrepreneurs are also safer, such contracts imply a strictly lower cost of credit for type-H entrepreneurs than for type-L ones.

3.2 Equilibrium separating contracts: outcome uniqueness

The unique outcome in terms of guarantees, probability of access to credit and cost of credit characterized in lemma 1 is associated with a unique equilibrium separating contract for type-H entrepreneurs. The same is not always true for type-L entrepreneurs. In particular, it is immediate to verify that for this type of borrowers, if $w-\eta > 0$, there is a continuum of contracts, characterized by collateral, $C_L \in [0, w-\eta]$, which yield the same outcome in terms of guarantees, $G_L = w - \eta$, and, therefore, of cost of credit and access to credit.[8] Therefore, strictly speaking, if $w - \eta > 0$, the

[7]Given equation (17), the derivative of π_H with respect to w yields

$$\frac{\partial \pi_H}{\partial w} = \begin{cases} \frac{\beta\left(Rp_L-(1+r)[p_L+\frac{1-p_L}{\beta}]\right)}{\{p_L R-\frac{p_L}{p_H}(1+r)-(1-p_L)\left[1-\frac{p_L}{p_H}\frac{1-p_H}{1-p_L}\beta\right]w\}^2} & \text{if } w \in (\eta, \eta+\frac{(1+r)}{\beta}) \\ \frac{[Rp_L-(1+r)](1-p_L)\left[1-\frac{p_L}{p_H}\frac{1-p_H}{1-p_L}\beta\right]}{\{p_L R-\frac{p_L}{p_H}(1+r)-(1-p_L)\left[1-\frac{p_L}{p_H}\frac{1-p_H}{1-p_L}\beta\right]w\}^2} & \text{if } w \leq \eta\ , \end{cases} \tag{20}$$

which is always strictly positive provided that assumption 1 holds. Note that the second-order derivative is also positive.

[8]Viceversa, C_L is uniquely determined and equal to zero, $C_L = 0$, if $w - \eta < 0$.

equilibrium separating contract designed for type-L entrepreneurs is not unique. However, this is true if and only if entrepreneurs do not incur any transaction cost to post collateral; otherwise, the contract characterized by $C_L = 0$ would dominate the others. We know that, in reality, such costs are always strictly positive. Because of that, and given that all equilibria yield the same outcome in terms of cost and access to credit and guarantees, we select the separating contract with $C_L = 0$ as the unique equilibrium separating contract with no loss of generality.

3.3 Pooling contracts

For a given level of wealth, w, pooling requires a contract, $\mathcal{C}_p = \{R_p^B, C_p, \pi_p\}$, such that,

1. Borrowers' participation constraints are satisfied:

$$(PC_H) \quad : \quad \pi[p_H(R - R_p^B) - (1 - p_H)G_p] \geq 0; \tag{21}$$

$$(PC_L) \quad : \quad \pi[p_L(R - R_p^B) - (1 - p_L)G_p] \geq 0. \tag{22}$$

2. Lenders make zero profits:

$$p_m R_p^B + (1 - p_m)G_p\beta = (1 + r), \tag{23}$$

where $p_m \equiv \mu p_H + (1 - \mu)p_L$, and G_p is the value of guarantees obtained from equation (1), given the value of R_p^B and C_p associated with the pooling contract. The following result holds,

Lemma 2 (Equilibrium pooling contracts: Characterization)**.** *In equilibrium, if pooling takes place among entrepreneurs endowed with wealth, w, the outcome is unique in terms of cost of credit, guarantees and access to credit. All the entrepreneurs demand credit, while lenders offer the following pooling contract $\mathcal{C}_p = \{R_p^B, C_p, \pi_p\}$, with $\pi_p = 1$, where*

i. If

$$\mu > \frac{\beta p_H(1 - p_L) - (1 - p_H)p_L}{[\beta p_H + (1 - p_H)]\,(p_H - p_L)}, \tag{24}$$

then,

$$R_p^B = \frac{(1 + r)}{p_m} - \frac{(1 - p_m)}{p_m}\beta \max\{w - \eta, 0\}, \tag{25}$$

$$C_p \in [0, w - \eta] \Rightarrow G_p = \max\{w - \eta, 0\}, \tag{26}$$

ii. If

$$\mu < \frac{\beta p_H(1 - p_L) - (1 - p_H)p_L}{[\beta p_H + (1 - p_H)]\,(p_H - p_L)}, \tag{27}$$

then,

$$R_p^B = \frac{(1+r)}{p_m} - \frac{(1-p_m)}{p_m}\beta \min\left\{w, \frac{1+r}{\beta}\right\}, \tag{28}$$

$$C_p = w \Rightarrow G_p = \min\left\{w, \frac{1+r}{\beta}\right\}, \tag{29}$$

iii. If

$$\beta\frac{1-p_m}{p_m} = \frac{1-p_H}{p_H}, \tag{30}$$

then, there is a continuum of equilibrium values of $C_p \in [0, w]$, $G_p = \max\{w - \eta, C_p\}$, *and correspondingly, of the interest rate, where*

$$R_p^B = \frac{1+r}{p_m} - \frac{1-p_m}{p_m}\beta G_p. \tag{31}$$

Proof. See appendix.

In equilibrium, if pooling takes place at the level of wealth, w, all entrepreneurs endowed with that level of wealth borrow under the same contract. No rationing takes place, and the cost of credit is decreasing in w. The value of guarantees, G_p, will depend on the level of collateral, C_p. Under perfect information, safe entrepreneurs would never find it convenient to post collateral to reduce credit cost, as repaying lenders using collateral is inefficient, given $\beta < 1$. However, if pooled with risky entrepreneurs, safe entrepreneurs would pay a higher cost of credit than in the perfect information case, effectively subsidizing risky entrepreneurs. Because of that, if pooled with risky entrepreneurs, they could find it convenient to post collateral to reduce credit cost. This happens if and only if condition (27) holds. In that case, the equilibrium pooling contract is characterized by maximum collateral, $C_p = w$, and maximum guarantees, i.e., $G_p = \min\{w, \frac{1+r}{\beta}\}$. Otherwise, if condition (24) holds, $C_p \in [0, w - \eta]$ and the equilibrium pooling contract is characterized by minimum guarantees, i.e., $G_p = \max\{w - \eta, 0\}$. Similarly to what discussed in subsection 3.2, also in this case the equilibrium outcome is not unique strictly speaking, in terms of collateral requirements. However, the same applies to what discussed in subsection 3.2 and the outcome is unique in terms of guarantees. Finally, in case iii, there exists a continuum of pooling equilibria outcomes, whereby entrepreneurs are indifferent about the level of collateral to post.

3.4 Credit market equilibrium

Having characterized pooling and separating equilibrium contracts for a given level of wealth, w, we now turn to their existence and characterize the credit market equilibrium. We study whether and under which conditions the equilibrium is characterized by separation or pooling for any level of wealth, $w \in [0, \overline{w}]$.

Proposition 1 (Credit market equilibrium)**.** *The credit market is characterized by a unique equilibrium outcome. In the absence of asset exemption, i.e., if $\eta = 0$, no separation occurs. With positive exemption, i.e., $\eta > 0$, there always exists a unique threshold value of wealth, $\widehat{w}_2 \in (0, \widehat{w}_1]$, such that:*

i. Poor entrepreneurs endowed with wealth, $w \leq \widehat{w}_2$ pool by means of the contract characterized in lemma 2;

ii. Entrepreneurs endowed with wealth, $w \geq \widehat{w}_2$ separate by means of the separating contract characterized in lemma 1 if the fraction of safe entrepreneurs in the population satisfies

$$\mu < \widehat{\mu}_1 \equiv \frac{p_H - p_L}{(1-\beta)p_H(1-p_H) + p_H - p_L} \tag{32}$$

Otherwise, if adverse selection is not severe, $\mu \geq \mu_1$, they pool according to the contract characterized in lemma 2.

Proof. See Appendix.

Separation requires lenders to be able to offer a menu of different contracts. In the absence of asset liquidation exemption, in the event of bankruptcy, independently of collateral requirements associated with the credit contracts signed by the entrepreneurs, lenders can appropriate entrepreneurial wealth up to $\max\{(1+r)/\beta, w\}$. Therefore, conditional on w, the level of guarantees cannot be different across contracts. Under these circumstances, the only way to achieve separation would be to differentiate contracts in terms of the probability of access to credit and the cost of credit. However, such a screening mechanism turns out to be ineffective as, in equilibrium, it would violate the single-crossing property. Therefore, no separation occurs without exemption. Differently, if there is positive asset exemption from liquidation, the level of real guarantees implied by a contract is affected by the collateral requirements specified by the contract. In particular, while a contract with no collateral requirements leads to guarantees equal to $\min\{\max\{w-\eta, (1+r)/\beta\}$, having collateral requirements can push the level of guarantees up to $\min\{w, (1+r)/\beta\}$. Accordingly, separation could be now achieved by offering a menu of contracts characterized by different guarantees induced by the different collateral requirements. Still, collateral is a costly screening device. This is because collaterizable assets are illiquid, in the sense that liquidation entails a loss of value. Hence, separation might take place only if the magnitude of the adverse selection effect on the cost of credit that type-H entrepreneurs face in a pooling equilibrium is strong enough. The strength of such adverse selection effect is inversely related to the fraction of type-H in the economy, μ, so that if μ is smaller than a critical value $\widehat{\mu}_1$, type-H entrepreneurs prefer to separate from type-L ones, while they would prefer to pool, otherwise. However, even when adverse selection is severe, sufficiently poor entrepreneurs of type-H still prefer pooling rather than separation. The reason is that being poor, they can separate from risky borrowers only by contracts characterized not only by higher collateral requirements but also by a very low probability of access to credit. This makes separation too costly to achieve even when adverse selection is severe. Finally, the lower is the liquid value of assets, i.e., the lower is the value of β, the more costly separation is. Accordingly, the minimum strength of the adverse selection effect needed for separation to occur, which is measured by $\widehat{\mu}_1$, is generally a function of β.

Corollary 1 (Adverse selection and liquidation value of the assets)**.** *If the fraction of safe entrepreneurs in the market, μ, satisfies*

$$\mu \leq \frac{p_H - p_L}{p_H(1-p_H) + p_H - p_L} \equiv \mu_2 \tag{33}$$

adverse selection is so severe that separation occurs in equilibrium irrespective of the liquidation value per unit of wealth, β. If $\mu > \mu_2$, adverse selection is severe enough for separation to occur if and only if the liquidation value per unit of wealth, β, is sufficiently high, so that $\mu \leq \widehat{\mu}_1$, holds. The higher the liquidation value per unit of wealth, β, the weaker the adverse selection effect must be for separation to occur.

Proof. See appendix.

Separation allows safe entrepreneurs to counteract the effects of adverse selection. However, as we know, separation through collateral requirements is costly for them; more so the lower is the liquid value of assets. Therefore, according to the above result, in markets in which the adverse selection effect is very severe, we should expect that safe borrowers prefer separation irrespective of the cost, that is, for any value of β. Differently, if the adverse selection effect is weaker because risky entrepreneurs are less frequent in the population, then we should expect separation only if the liquidation value of the assets is sufficiently high, i.e., β is sufficiently close to 1, which makes separation less costly to achieve.

4 Effects of asset exemption from liquidation under Chapter 7

Having characterized the equilibrium in the credit market, we now discuss the effects of debtor protection in the form of asset exemption from liquidation upon default on access to and the cost of credit. In order to do so, we analyze how a change in the level exemption, η, affects the equilibrium outcome. We already know from proposition 1 that without exemption, no separation occurs. Under these circumstances, the credit market is characterized by no credit rationing and no dispersion in lending rates. With positive exemption, so long as adverse selection is sufficiently severe, separation occurs, and credit rationing can emerge. The effect of an increase in asset exemption on credit rationing depends on the interaction of the exemption level with the wealth distribution of entrepreneurs of type-H who are the candidates to self-select in contracts characterized by collateral requirements and, possibly, credit rationing. It is immediate to verify that, according to equation (19), the critical level of wealth such that type-H entrepreneurs are rich so that they can separate without being rationed, $\widehat{w}_1$, is decreasing in the level of exemption for levels of exemption sufficiently low and stays constant otherwise. Through this channel, a higher exemption level should reduce credit rationing, if anything. Moreover, when the level of exemption is sufficiently low, for any given level of wealth, $w < \widehat{w}_1$, a higher η increases the probability of having access to credit for poor entrepreneurs of type-H who separate and are rationed, π_H,

$$\left.\frac{\partial \pi_H}{\partial \eta}\right|_{\eta<\overline{\eta}} = \frac{(1-p_L)(1-\beta)}{p_L R - \frac{p_L}{p_H}(1+r) - (1-p_L)\left[1 - \frac{p_L}{p_H}\frac{1-p_H}{1-p_L}\beta\right] w} > 0. \tag{34}$$

Note also that the cross derivative with respect to w is positive, such that the effect becomes more relevant as w increases. Also, through this channel, an increase of exemption should reduce credit rationing, if anything. However, an increase in the level of exemption might also reduce the minimum amount of wealth, $\widehat{w}_2$, such that poor entrepreneurs of type-H endowed with $w \geq \widehat{w}_2$ choose to separate if adverse selection is sufficiently severe.[9] Through this channel, more entrepreneurs would become rationed as exemption increases. The sign of the overall effect of exemption on aggregate credit rationing is in principle ambiguous and depends on the shape of entrepreneurial

[9] Taking the total differential of the difference between the equilibrium entrepreneur's payoffs under separation and pooling, with respect to $\widehat{w}_2$ and η, and setting it equal to zero yields,

$$\left.\frac{d\widehat{w}_2}{d\eta}\right|_{\eta<\overline{\eta}} = \frac{(1-p_H)\left(1-\frac{(1-p_m)p_H}{p_m(1-p_H)}\beta\right) - \frac{(1-p_L)(1-\beta)[p_H R-(1+r)-(1-p_H)(1-\beta)w]}{p_L R - \frac{p_L}{p_H}(1+r)-(1-p_L)\left(1-\frac{p_L(1-p_H)}{p_H(1-p_L)}\beta\right)w}}{\pi_H\left[\lambda_{ICC}((1-p_L)-\frac{p_L}{p_H}(1-p_H)\beta)-(1-p_H)(1-\beta)\right]+(1-p_H)\left(1-\frac{p_H(1-p_m)}{p_m(1-p_H)}\beta\right)} \tag{35}$$

which is strictly negative. On the contrary, $\frac{d\widehat{w}_2}{d\eta} = 0$ for $\eta \geq \overline{\eta}$.

wealth distribution. If most of the entrepreneurs have wealth, w, greater than $\widehat{w}_2$, we should expect that an increase in η has a negative effect on credit rationing. On the contrary, if most of the entrepreneurs have wealth, $w \leq \widehat{w}_2$, we could expect more rationing. However, an exemption increase reduces the probability of rationing for type-H entrepreneurs who separate, as discussed above.

The level of exemption also affects the cost of credit differential between the contract designed for type-H entrepreneurs and that designed for type-L ones. Calculating the difference between R_L^B and R_H^B and taking the derivative with respect to η yields

$$\frac{\partial(R_L^B - R_H^B)}{\partial\eta} = -\frac{1-p_L}{p_L}\beta\frac{\partial G_L}{\partial\eta} + \frac{1-p_H}{p_H}\beta\frac{\partial G_H}{\partial G_L}\frac{\partial G_L}{\partial\eta}. \tag{36}$$

We know that the derivative of G_L with respect to η is zero if $\max\{0, w-\eta\} = 0$, which happens under sufficiently high (low) levels of η (w), and positive if $\max\{0, w-\eta\} = w-\eta$, which is the case for sufficiently low (high) levels of η (w). Moreover, we know from the expression of G_H that the derivative of G_H with respect to G_L is less than one. Hence, given $p_H > p_L$, the above derivative is zero when $\max\{0, w-\eta\} = 0$ and positive if $\max\{0, w-\eta\} = w-\eta$.

The above results allow us to provide an assessment of the welfare and distributional effects of asset exemption policy provision under Chapter 7 of the US personal bankruptcy law in a market characterized by adverse selection. To the extent that exemption might induce a separating equilibrium, it improves the welfare of safer entrepreneurs at the expenses of riskier ones, as it cuts the chances that the latter can benefit from the implicit subsidised cost of credit associated with pooling. The equilibrium contract, whether separating or pooling, is always the one that maximizes safe entrepreneurs' payoff. Therefore, separation, when it occurs, makes entrepreneurs better off compared to pooling, which would be the only option under no exemption. However, this comes at the cost of underinvestment that is associated with credit rationing, which is inefficient and therefore constitutes an undesirable feature of the exemption policy.

5 Empirical analysis

We now take the model to the data to provide an empirical assessment of the effects of the asset liquidation exemption policy. First, we review the empirical implications of the model. Then, we discuss the econometric specifications we use for testing the model's predictions, and finally, we present the empirical results.

5.1 Empirical Implications: Cost of credit, access to credit and exemption

Given the equilibrium characterization provided in sections 3 and 4, the model delivers the following empirical implications:

1. **Cost of credit.** First, the cost of credit is negatively associated with entrepreneurial wealth. Second, under positive asset exemption, the cost of credit is negatively affected by the decision to post collateral.

2. **Access to credit.** Under positive asset exemption, the following holds. First, entrepreneurs' decision to post collateral is associated with a lower probability of accessing credit. Second, within the group of entrepreneurs posting collateral, poorer entrepreneurs are more likely to be rationed than richer ones.

3. **Effects of exemption.** First, a higher level of exemption is associated with a higher cost of credit differential in favor of entrepreneurs who post collateral, compared to those who do not post any. Second, conditional on wealth, for those entrepreneurs posting collateral, the probability of being rationed declines with exemption.

5.2 Data

We use the publicly available version of the 2003 wave of the Survey of Small Business Finances (SSBF), conducted in 2004-05 for the Board of Governors of the Federal Reserve System. We stick to this survey because it has been widely employed in the literature, which improves our results' comparability. Relevant to our analysis, Berger, Cerqueiro, and Penas (2011) and Berkowitz and White (2004) both study the relationship between exemption and access to credit using the SSBF data.[10] The data provide information on a sample of 4240 firms, selected from the target population of all for-profit, non-financial, non-farm, non-subsidiary business enterprises that had fewer than 500 employees and were in operation as of year-end 2003 and on the date of the interview. Information on the availability and use of credit and other financial services, demographic characteristics for up to three of the individual owners, and other firm characteristics such as the number of workers, organizational form, location, credit history, income statement, and balance sheet is available. The survey asked entrepreneurs whether their firm applied for credit during the last three years

[10] Berger, Cerqueiro, and Penas (2011) combine various waves of the same survey over 1996-2005, while Berkowitz and White (2004) use the 1993 wave.

(from 2001 to 2003) and, if so, whether such applications were always denied, always approved, or sometimes approved.

Our estimation strategy is to use the model as an identification tool. Accordingly, since all applicants are creditworthy in our model, we restrict our sample to those firms whose loan applications have been approved at least once in the observation period. By doing so, the sample size is reduced to 1761 firms, 96% of which were always financed. For the most recent loan contract signed by each of these firms in 2001-03, the survey provides information on the interest rate and whether the firm had to post collateral.[11] According to the model, only type-H firms post collateral. Therefore, we use the decision to post collateral to identify a firm's type in the data. We are aware that focusing on creditworthy applicants might cause a selection bias, and – as detailed in subsection 5.6– after presenting our main empirical results with run a robustness check in that respect.

5.2.1 Measures of exemption and entrepreneurial wealth

We augment the SSBF data by including the level of bankruptcy homestead and personal property exemptions according to the firm's geographical location. Exemption levels vary across states, but unfortunately, the public version of the SSBF reports the firm's location only for the nine census divisions (New England; Middle Atlantic; East North Central; West North Central; South Atlantic; East South Central; West South Central; Mountain; Pacific). Thus, the best we can do is exploit exemption variability across census divisions rather than states by assigning the average level of exemption of its census division to each firm. Furthermore, determining the average level of exemption per census division is not trivial due to the existence of states with an unlimited exemption. Fortunately, most of the states in which exemption is unlimited concentrate on two of the nine census divisions: West North Central and West South Central. Accordingly, we construct a dummy variable that takes value one (high exemption) for firms located in these two census divisions and zero (low exemption) otherwise.[12] The firm's total assets measure its wealth. We define two groups of firms. A *high assets* group includes firms with asset values above the median, and a *low assets* group includes firms with asset values below the median. Thus, based on wealth and exemption, we ultimately have four categories of firms.

[11] The dataset does not provide information on the amount of collateral posted.

[12] Alternatively, we could have computed the average exemption per census division by assigning each state with unlimited exemption a value of exemption equal to the average dollar value of the assets of firms located in the state's census division. Following this alternative procedure would deliver qualitatively the same results as those we obtained.

5.3 Descriptive statistics

Table 1 reports the descriptive statistics on the cost of credit and the probability of being rationed, both for the entire sample and for the relevant subsamples. The observed patterns are as follows:

1. High-asset firms face a lower cost of credit and a lower probability of being rationed. For these firms, the loan rate and the fraction of rationed firms are 1.5 and 3.8 percentage points lower than for firms in the low-asset subsample, respectively;

2. Firms that post collateral face a lower cost of credit. These firms face a loan rate that is 0.7 percentage points lower than that charged to other firms. This effect is larger the higher the exemption level is. In the low-exemption subsample, the cost of capital differential in favor of firms posting collateral is 0.53%, while that in the high-exemption subsample grows to 1.20%;

3. The correlation between the decision to post collateral and the cost of credit depends on wealth. Low-asset firms gain a reduction of 0.9 percentage points in the cost of credit if posting collateral. For high-asset firms, the corresponding reduction is much smaller (0.04%);

4. Firms that post collateral face a higher probability of rationing. The fraction of rationed firms in the subsample of firms posting collateral is 1.5 percentage points above the correspondent fraction in the subsample of firms that do not post collateral.

5. The association between rationing and collateral depends on wealth. In the subsample of low-asset firms, the fraction of rationed firms is 4.4% higher for firms that post collateral compared to those that do not. In the high-asset subsample, there is no difference in rationing depending on collateral;

6. Among firms posting collateral, the fraction of those that are rationed falls by 1.1% when moving from low to high exemption levels. This effect is larger (-1.9%) for low-asset firms compared to high-assets ones (-0.5%).

Notably, the above evidence is entirely consistent with our model. In particular, (a) the loan rate differential in favor of firms posting collateral grows with exemption; (b) a smaller fraction of firms posting collateral is rationed in high-exemption census divisions than in low-exemption census divisions. We now proceed to test the model's key predictions.

5.4 Cost of credit and access to credit: econometric specifications

In this subsection, we derive the econometric specifications for cost of credit and access to credit.

Cost of credit. Our theory predicts that in a separating equilibrium, entrepreneurs posting collateral – who are type type-H – face a lower cost of credit. In the data, we only observe the cost of credit associated with the decision to post collateral, which we call R_H^B following our model. We do not observe the counterfactual cost of credit that firms would have paid had they not posted collateral, which we refer to as $R_L'^B$. Similarly, while we observe the cost of credit faced by firms that do not post collateral, R_L^B, we do not observe the counterfactual cost of credit that they would have faced had they posted collateral, $R_H'^B$. To circumvent this issue, we construct the counterfactual values of the cost of credit, $R_L'^B$ and $R_H'^B$, using an endogenous switching approach (Maddala, 1986), under the identifying assumption – based on our theory that the observed values of the cost of credit are associated with the decision to post collateral. Accordingly, we model the observed cost of credit, measured by the loan rate, for the two subsets of firms that self-select according to their collateral decision, $C = \{0, 1\}$, where 1 means "posting collateral" and 0 means "not posting collateral":

$$R_i^B|C = \mathbf{X_i}\beta + u_i. \tag{37}$$

The endogenous switching approach allows us to account for firms' self-selection by modeling the decision to post collateral and linking this decision to the cost of credit. The benefit of posting collateral has the following empirical specification

$$K_i^* = \mathbf{Z_i}\gamma + v_i, \tag{38}$$

where K^* is the net benefit of posting collateral, $\mathbf{Z}$ is a set of explanatory variables, γ is a vector of parameters, and v is the error term. Therefore, the decision of firm i to post collateral is:

$$C_i = \begin{cases} 1 \quad if \;\; \mathbf{Z_i}\gamma + v_i > 0, \\ 0 \quad if \;\; \mathbf{Z_i}\gamma + v_i \leq 0. \end{cases} \tag{39}$$

To correctly estimate equation (37), we need to consider the latent variables that affect the decision to post collateral. More precisely, given the self-selection model (39), assuming that u and v are bivariate normal, the expected value of $R_i^B|C$ is as follows:

$$E(R_{L,i}^B|C = 0) = \mathbf{X_i}\beta_{1_L} - \sigma_{1_{L,v}} \frac{\phi(-\mathbf{Z_i}\gamma)}{\Phi(-\mathbf{Z_i}\gamma)}, \tag{40}$$

$$E(R^B_{H,i}|C=1) = \mathbf{X_i}\beta_{1_H} + \sigma_{1_{H,v}} \frac{\phi(-\mathbf{Z_i}\gamma)}{1-\Phi(-\mathbf{Z_i}\gamma)}, \tag{41}$$

where ϕ is the pdf of the standard normal distribution, and Φ is the cumulative density function.[13] The functions $\lambda_{L,i} = -\frac{\phi(-\mathbf{Z_i}\gamma)}{\Phi(-\mathbf{Z_i}\gamma)}$ and $\lambda_{H,i} = \frac{\phi(-\mathbf{Z_i}\gamma)}{1-\Phi(-\mathbf{Z_i}\gamma)}$ are the inverse Mills ratios, and they represent the conditional expectation of v given the selection into not posting or posting collateral, respectively; that is, $\lambda = E(v_i|C)$.[14] Regarding the expected values of the unobserved cost of credit, following Maddala (1986), we have

$$E(R'^B_{L,i}|C=0) = \mathbf{X_i}\beta_{2_H} - \sigma_{2_{H,v}} \frac{\phi(-\mathbf{Z_i}\gamma)}{\Phi(-\mathbf{Z_i}\gamma)}, \tag{42}$$

which is the expected cost of credit faced by firms posting collateral had they chosen not to post it, and

$$E(R'^B_{H,i}|C=1) = \mathbf{X_i}\beta_{2_L} + \sigma_{2_{L,v}} \frac{\phi(-\mathbf{Z_i}\gamma)}{1-\Phi(-\mathbf{Z_i}\gamma)}, \tag{43}$$

which is the expected cost of credit for those not posting collateral had they chosen to post collateral.

The overall estimation procedure is as follows. First, we obtain the appropriate inverse Mills ratios by estimating the selection process (equation 39) using the following probit specification:

$$C_i = \mathbf{Z_i}\gamma + v_i, \tag{44}$$

where the linear predictions, $\mathbf{Z_i}\widehat{\gamma}$, that we obtain by estimating (44) are used to compute the estimated values of the inverse Mills ratios. Then, based on equations (40-43), we estimate the loan rates with OLS. Finally, we compute the estimates of the expected loan rates. For firms not posting collateral, we have

$$\widehat{R}^B_{L,i} = \mathbf{X_i}\beta_{1_L} - \sigma_{1_{L,v}}\lambda_{L,i}, \tag{45}$$

$$\widehat{R}'^B_{H,i} = \mathbf{X_i}\beta_{2_L} + \sigma_{2_{L,v}}\lambda_{H,i}. \tag{46}$$

where $R'^B_{H,i}$ is the counterfactual interest rate. Similarly, for firms posting collateral, we obtain

$$\widehat{R}^B_{H,i} = \mathbf{X_i}\beta_{1_H} + \sigma_{1_{H,v}}\lambda_{H,i}, \tag{47}$$

$$\widehat{R}'^B_{L,i} = \mathbf{X_i}\beta_{2_H} - \sigma_{2_{H,v}}\lambda_{L,i}. \tag{48}$$

[13]The results of equation (40) follow due to the truncation of the distribution of R^B_L from above: $E(R^B_{L,i}|C = 0) = E(R^B_{L,i}|v_i \leq -\mathbf{Z_i}\gamma) = \mathbf{X_i}\beta_{1_L} + E(u_L|v_i \leq -\mathbf{Z_i}\gamma) = \mathbf{X_i}\beta_{1_L} - \sigma_{1_{L,v}}\frac{\phi(-\mathbf{Z_i}\gamma)}{\Phi(-\mathbf{Z_i}\gamma)}$. The results of equation (41) follow from the truncation of R^B_H from below: $E(R^B_{H,i}|C = 1) = E(R^B_{H,i}|v_i > -\mathbf{Z_i}\gamma) = \mathbf{X_i}\beta_{1_H} + E(u_H|v_i > -\mathbf{Z_i}\gamma) = \mathbf{X_i}\beta_{1_H} + \sigma_{1_{H,v}}\frac{\phi(-\mathbf{Z_i}\gamma)}{1-\Phi(-\mathbf{Z_i}\gamma)}$.

[14]Note that v_i is the part of K_i not explained by the observable information represented by the $\mathbf{Z_i}$ explanatory variables. In this sense, v_i is the private information that influences the decision of whether to post collateral. Ex ante, $E(v_i) = 0$, but ex-post, after the firm decides whether to post collateral, the expectation on v_i can be updated. $E(v_i|C)$ is the updated estimate of firm private information (Kai and Prabhala, 2007).

Note that the model is identified by the non-linearity inherent in the inverse Mills ratio. Based on our theory, we expect that the estimated parameters are $\hat{\sigma}_{1_{L,v}} < 0$ and $\hat{\sigma}_{1_{H,v}} < 0$. By posting collateral, type-H firms self-select into contracts characterized by a cost of credit lower than the average. Conversely, type-L firms that self-select into contracts characterized by no collateral requirements face a cost of credit that is above average.

This approach considers the endogeneity arising from the simultaneous determination of the cost of credit and collateral and the role of private information implicit in the decision to post collateral.[15] According to our theory and as suggested by Kai and Prabhala (2007), in the self-selection model in equations (40)-(43), the decision to post collateral captures some unobserved heterogeneity about firm's type. By posting collateral, firms reveal private information about their type, which affects the cost of credit that they will face, through the parameters $\sigma_{1_{L,v}}$ and $\sigma_{1_{H,v}}$. In summary, we have the following:

1. The statistical significance of the coefficient associated with the inverse Mills ratio captures the self-selection effects associated with the choice of posting collateral;

2. The sign of the coefficient of the inverse Mills ratios identifies the benefit in terms of the cost of credit for those firms that post collateral compared to those that do not post any; and,

3. The variables λ_L and λ_H estimate the private information underlying firm's choice. Hence, the statistical test of their significance is a test of whether private information possessed by the firm explains ex-post results (cost of credit) (Kai and Prabhala, 2007).

Access to credit. Applying our theoretical model, to the extent that safe firms separate from risky ones, the structural form equation of the probability of access to credit, π, is (see (16) and (17), proposition 1) is:

$$\pi = \begin{cases} \min\left(\frac{p_L(R-R_L^B)-(1-\beta)w_\eta}{p_L(R-R_H'^B)-(1-p_L)G_H}, 1\right) & \text{for safe firms} \\ 1 & \text{for risky firms ,} \end{cases} \tag{49}$$

where $R_H'^B$ is the (counterfactual) cost of credit that risky firms would have paid had they posted collateral. In contrast, to the extent that heterogeneous firms pool together, $\pi = 1$ holds (see proposition 2). Accordingly, the probability of access to credit is i. (weakly) decreasing as we move from risky firms to safe ones, as only poor entrepreneurs of type-H, if any, are rationed according

[15] An alternative approach to account for this endogeneity is to estimate a simultaneous model of joint determination of collateral and the cost of credit. We employ this alternative approach as a robustness check in section 5.6.

to the model; ii. increasing in the level of firm wealth; and iii. decreasing in the cost of credit. Moreover, the effect associated with a firm's type is declining with exemption, as the model implies that type-H entrepreneurs are less rationed the higher the level of exemption is. Significantly, according to the model, both the cost of credit and a firm's type are exogenous with respect to the probability of having access to credit. Accordingly, we specify the following econometric model for the probability of firm i having access to credit:

$$\pi_i = \alpha_1 \mathbf{Y_i} + \alpha_2 \eta_i + \alpha_3 C_i + \alpha_4 C_i \times \eta_i + \alpha_5 \frac{R^B_{L,i}}{R'^B_{H,i}} + u_i, \tag{50}$$

where π_i takes two values, 1 if firm's loan applications have always been approved and 0 if they have only sometimes been approved; $\mathbf{Y_i}$ is a set of controls that affect a bank's decision to supply credit; $\boldsymbol{\alpha_1}$ is a vector of parameters; α_2, α_3, α_4 are parameters; η_i is a dummy that equals one if the firm is located in a census division with high exemption; C_i is a dummy that equals one if firm i posts collateral; $C_i \times \eta_i$ is an interaction term; and $u_i \sim N(0, \sigma_1)$ is the error term. We estimate equation (50) by probit.

Following our theoretical model, the variable C_i captures a firm's type, as only safe firms (type-H) post collateral, while the interaction term captures the model's prediction according to which access to credit should improve for safe firms, which post collateral, compared to risky firms (type-L), which do not, as exemption increases. The loan rates R^B_L and R'^B_H are proxied by the predicted values resulting from the estimation of equations (45)-(46). In line with our theory, see (equation 49), we include the ratio between the actual and the counterfactual loan rates for risky firms in equation (50). This ratio can also be viewed as an indicator of the relevance of the private information revealed by the decision not to post collateral. Our estimation differs from the model estimated in Berkowitz and White (2004) as we take into account the simultaneity of the cost of credit and collateral decisions, as well as the extent of private information in access to credit. According to our theory, we expect $\hat{\alpha}_3 < 0$, $\hat{\alpha}_4 > 0$ and $\hat{\alpha}_5 < 0$.

5.4.1 Control variables

The sets of controls $\mathbf{X_i}$, $\mathbf{Z_i}$ and $\mathbf{Y_i}$ in models (44)-(48), and (50) contain variables related to firm-characteristics that have been found to have a significant impact either on the probability of accessing credit, the cost of credit or both, in the empirical literature.

Sorensen and Chang (2006) provide evidence of a positive relationship between an entrepreneur's experience and the firm's profitability. We capture entrepreneurs' experience, including the number of years of managerial experience held by the principal owner.

Belonging to a minority group has been found to reduce the probability of obtaining a loan (Cavalluzzo and Wolken, 2005; Berkowitz and White, 2002). Cerqueiro and Penas (2017), find evidence that owners belonging to a minority group rely more heavily on their own funds to finance a startup. We control for minorities by means of two dummies. The first takes value 1 if the principal owner is black and 0 otherwise. The second takes value 1 if the owner belongs to other minority groups (asian, hispanic, asian pacific, native american) and 0 otherwise. We also include a dummy indicating whether the owner is female, to assess possible discrimination effects. A firm's proprietorship characteristics may affect access and cost to credit, as family ownership may reduce agency costs and promote trust. Anderson, Mansi, and Reeb (2003) suggest that if families tend to maintain a favorable reputation with the firm's debt holders, we should observe a negative relationship between family proprietorship and the cost of credit. Niskanen, Niskanen, and Laukkanen (2010) find evidence that family ownership is associated with lower availability of credit for small Finnish firms, while managerial ownership leads to lower collateral requirements.[16]

The firm-bank relationship can be represented by several variables, such as the firm's distance from the bank and the length of the relationship with the lender. The structure of local credit markets may also have a role in explaining the cost of credit. To account for banks local market power, we include a dummy that is equal to one if the Herfindahl-Hirschman bank deposit index of local credit market concentration is greater than 1800.[17] We also include the number of credit applications in the previous three years as a proxy for a firm's financial needs.[18] To control for a borrower's observable quality, we include a dummy that is equal to one if the firm's credit score is in the top 25% of the distribution. We also account for the fact that the cost of the loan might be affected by loan characteristics. Accordingly, we distinguish two typologies: 1. line of credit and 2. fixed interest rate loans. Finally, we control for firm's scale using the log of sales, and we use the ratio of debt to total assets as a measure of a firm's financial structure, i.e., the firm's leverage. Finally, as mentioned above, a firm's wealth is proxied by its assets. We also include controls specific to each of the econometric models. In the model of the decision to post collateral (equation (44)), we employ as controls the dummy for high credit score (top 25%), loan maturity, the amount granted over the total amount applied for, bank market concentration, a dummy for limited liability,

[16]They suggest that family ownership increases agency costs, which the bank accounts for when dealing with such firms.

[17]In the public version of the SSBF, bank market concentration is reported in three classes: Herfindahl index below 1000, between 1000 and 1800, and above 1800.

[18]Frequent loan applications may be a signal of either financial distress or greater investment opportunities.

a dummy for a female applicant, the length of firm-bank relationships, and the dummy for family proprietorship. In the econometric model for the probability of access to credit (equation (50)), the control variables are mainly related to loan characteristics and firm-bank relationships. We consider the funds granted over the total amount requested. Larger loans, given other firm and loan characteristics, increase bank profits and the bank's willingness to finance. We also include loan maturity, which we expect to harm the probability of having access to credit, as long-term loans could be less liquid and, therefore, more risky from the bank's perspective. A longer firm-bank relationship improves the information flow between lenders and borrowers. We include the number of years of the relationship with the lender, and we expect it to affect the probability of receiving a loan positively. Past delinquencies may represent a bad signal regarding firm trustworthiness. Thus, we expect a negative sign for the dummy that equals one if the firm has a delinquency record. As in the loan rate equation, we include a firm's credit score to proxy for its credit quality. We also control for a firm's capital structure. The ratio of debt to total assets is expected to harm the bank's willingness to finance because higher leverage may reduce the firm's ability to repay. A firm's wealth, proxied by its assets, is expected to affect the probability of having access to credit positively. Finally, we include a dummy that is equal to one if the firm has limited liability, which might limit banks' ability to seize owners' wealth in the event of default.

5.5 Cost and access to credit: empirical results

Cost of credit. In table 4, we report the estimation of the expected cost of credit.[19] Our results show that the information conveyed by the collateral decision is relevant. As predicted by our theory, the coefficients of the inverse Mills ratios are negative and significant. The negative signs of the coefficients of λ_H and λ_L imply a negative correlation between the unexplained factors that affect the cost of credit and those that affect the decision to post collateral. This means that other things being equal, the decision to post collateral implies a lower cost of credit. Firms that post collateral (type-H firms) have a below-average expected cost of credit. Also, for firms that do not post collateral (type-L firms) expected cost of credit would be lower had they posted collateral. It is also worth noting that the estimated $\hat{\sigma}_{1L,v}$ is the double of $\hat{\sigma}_{1H,v}$, meaning that type-H firms, which post collateral, choose contracts involving an expected cost of credit with lower variance.

As predicted in the theoretical model (equation 13), an increase in exemption raises the cost of

[19] In table 2 we report the estimates of the ancillary probit regression of the probability to post collateral (equation 44)

credit, R_L^B, for those firms not posting collateral, while the opposite is the case for firms posting collateral. The cost of credit faced by these firms, R_H^B, decreases with asset exemption. Finally, an increase in wealth (as proxied by the level of firm assets) reduces the cost of credit for firms not posting collateral, R_L^B, but only in high-exemption areas. Wealth also reduces the cost of credit for borrowers posting collateral, R_H^B, but not in high-exemption areas: in these areas, we find no significant effect of wealth on R_H^B. This is also consistent with our theory, according to which firms could undo the effect of exemption by posting collateral.

In summary, the above empirical results are entirely consistent with the theoretical model we propose because

i. the collateral decision conveys private information about firm type;

ii. posting collateral involves a lower cost of credit in high-exemption areas, and

iii. exemption is negatively correlated with the cost of credit faced by firms posting collateral, R_H^B, and positively correlated with the cost of credit of firms not posting collateral, R_L^B.

Access to credit. In table 3, we report the results of the estimation of the model (50) for the probability of access to credit. In column 2, we report the estimation results obtained with the estimation method that accounts for the possible bias due to the fact that the SSBF dataset is imputed.[20] The procedure increases the variance of the parameters and may reduce their statistical significance. The firm's type, as proxied by the decision to post collateral, C_i, and the interaction term between the decision to post collateral and the high exemption dummy are highly significant and with the expected sign. Posting collateral is positively associated with rationing. However, firms that post collateral are less likely to be rationed if they are located in a census division with a high exemption. Also, consistent with our theoretical model, an increase in the cost of credit faced by firms posting collateral increases their probability of access to credit. An increase in the cost of credit faced by firms not posting collateral reduces the probability that firms posting it will have access to credit.

Our estimates cannot be easily reconciled with the intuition according to which the decision to post collateral, by increasing the guarantees, should be associated with higher access to credit and a lower cost. Conversely, the evidence we find, according to which firms posting collateral face a

[20] In particular, following Rubin, 1987, we adopt an estimation procedure that computes estimates of coefficients and standard errors by applying combination rules to the individual estimates obtained by each imputation. This is implemented in STATA by means of the `mi estimate` command.

lower cost and lower access to credit, is consistent with our theory based on private information about firms' type and the possibility that firms self-select into different contracts based on their type. In conclusion, the main predictions of our theoretical model regarding access and cost of credit and the decision to post collateral and the effects of exemption cannot be rejected. Our results complement those found in the literature. Similar to Gropp, Scholz, and White (1997); Berkowitz and White (2004); Berger, Cerqueiro, and Penas (2011) we find a positive association between (i) exemption and rationing and (ii) exemption and the cost of credit. Our contribution is to show that according to the data, while the effect of high exemption is to increase credit rationing, the interaction between collateral and exemption tells us that, conditional on posting collateral, rationing is comparatively lower in high-exemption areas. That is, we cannot reject our model's hypothesis that posting collateral should be associated with a reduction in the probability of being rationed as exemption increases due to the enhanced power of the decision to post collateral as a sorting device. Similarly, we show that the negative effect of the decision to post collateral on the cost of credit grows in magnitude with the level of exemption. Thus, the evidence is consistent with the fact that collateral plays a role as a screening device and is a signal of quality, as in Jimenez, Salas, and Saurina (2006).

5.6 Robustness checks

Collateral, exemption and cost of credit. An alternative methodology to test our theoretical model regarding the impact of the decision to post collateral on the cost of credit, depending on the exemption level, would be to use the following reduced-form:

$$R_i^B = \boldsymbol{\beta_1}\mathbf{X_i} + \beta_2\eta_i + \beta_3 C_i + \beta_4 C_i\eta_i + v_i, \tag{51}$$

where $\mathbf{X_i}$ is a set of controls; $\boldsymbol{\beta_1}$ is a vector of parameters; η_i is a dummy that equals one if a firm is located in a census division with high exemption; C_i is a dummy that equals one if firm i posts collateral; $C_i \times \eta_i$ is an interaction term; β_2, β_3, β_4 are parameters; and $v_i \sim N(0, \sigma_2)$ is the error term. We estimate the loan rate equation by OLS. Our theory predicts that firms posting collateral (type-H firms) face a lower cost of credit. Hence, we expect that $\hat{\beta}_3 < 0$. Furthermore, the cost of credit differential in favor of firms of type-H should increase with exemption, such that we expect that $\hat{\beta}_4 < 0$. Finally, we expect that $\hat{\beta}_2 > 0$, as our theory predicts that the interest rate increases in exemption for firms not posting collateral (type-L firms).

For robustness, we estimate the above model and report the results in table 5.[21] Both the coefficient for firm type, identified by the decision to post collateral, C_i, and that for the interaction term between the decision to post collateral and the high exemption dummy, are significant and of the expected sign. This alternative estimation approach confirms the evidence that firms that post collateral face a lower cost of credit, and this effect grows with the level of exemption, in line with the predictions from the theoretical model. On average, firms posting collateral pay 0.30% less per unit of loans compared to firms that do not post collateral. Moving from a state with low exemption to a state with high exemption, posting collateral increases the discount by 0.55%.

Simultaneity between the cost of credit and guarantees. According to the model, the equilibrium levels of firm guarantees conditional on the entrepreneur's type, G_L, and G_H, and the corresponding values of the cost of credit, R_H^B, and R_L^B – none of which are affected by the probability of having access to credit – are simultaneously determined. Therefore, for robustness, we also estimate a system of two equations for the cost of credit as a function of the guarantees and the amount of guarantees as a function of the cost of credit.[22] The estimation results (table 6) show a negative relationship between R^B and G. Other things being equal, posting guarantees is associated with an average reduction in the cost of credit of 34 basis points. For the subsamples of the firms located in groups of states with high exemption levels, the reduction in the cost of credit associated with posting guarantees is three times larger than that for firms located in low-exemption states (75 vs 17 basis points). On overall, this alternative methodology confirms the empirical evidence we found using the estimation approach presented in the previous subsections.

Selection. As mentioned above, following our theoretical model, we conduct our empirical analysis on the subsample of creditworthy firms demanding credit. This leads to the possibility of a sample selection bias because firms applying for credit and creditworthy firms are selected subsamples. We control for this possibility by estimating a selection model à la Heckman. Notably, controlling for selectivity does not alter our conclusions as it can be seen from the estimation results of the cost of credit and of the probability of rationing reported in tables 7 and 8.[23]

[21] Note that in the SSBF survey, most of the missing variables have been originally imputed employing a randomised regression model. Accordingly, we take into account the possible bias in the estimation arising from multiple imputations. A more detailed discussion of data imputation in the SSBF can be found in the 2003 Technical codebook available at http://www.federalreserve.gov/pubs/oss/oss3/ssbf03/codebook/codebook03.pdf. In Column (2) of table 5, we report the estimation corrected to account for bias due to data imputation.

[22] Details on the estimation methodology are available upon request.

[23] Details on the implementation of the estimation procedure are available upon request.

6 Conclusion

According to the US personal bankruptcy law, under Chapter 7 entrepreneurs benefit from a debtor protection scheme because of asset exemption from liquidation upon default. The exemption, however, does not apply in case of debt secured by collateral. We show that the fact that entrepreneurs can undo the effect of asset exemption by posting collateral has significant implications for the equilibrium outcome in a competitive credit market characterized by adverse selection. With positive levels of asset exemption, collateral becomes an effective screening device. Accordingly, if adverse selection is severe, safer applicants separate from riskier ones if sufficiently wealthy by self-selecting in contracts characterized by higher collateral requirements. Differently, pooling prevails if there is no exemption or if adverse selection is not severe. In separating equilibria, the decision to post collateral reduces the cost of credit and, for wealth-constrained applicants, is associated with credit rationing. Because of that, sufficiently safe poor entrepreneur always pool irrespectively of adverse selection. As the level of asset exemption increases, access to credit for wealth-constrained applicants posting collateral is enhanced, and credit cost is further reduced. The empirical analysis we conduct based on the SSBF data support the predictions of our model. Our results inform about the welfare and distributional effects of the asset liquidation exemption policy. In distributional terms, safer entrepreneurs benefit from asset exemption as it allows for separating equilibria, which makes them better off. Correspondingly, riskier types are worse off when separation is the equilibrium outcome, as they no longer benefit from an implicitly subsidised cost of credit. However, asset exemption also produces credit rationing, which results in inefficient underinvestment.

A Appendix

A.1 Proof of lemma 1

We characterize the equilibrium separating contracts under the assumption that entrepreneurs disclose their wealth when borrowing. In section A.3, we prove that indeed they do so.

i. Cost of credit. The following preliminary result holds:

Lemma 3. *In any equilibrium, lenders must be making zero profits.*

Proof. Consider a candidate equilibrium in which, given the set of contracts offered by lenders and demanded by entrepreneurs with positive probability, lenders make strictly positive profits. We have to consider two cases. The case in which such set of contracts includes a pooling contract, $\mathcal{C}_p$ (case a), and that in which it contains separating contracts, $\mathcal{C}_H, \mathcal{C}_L$, with $\mathcal{C}_H \neq \mathcal{C}_L$ (case b). We note that in both cases, a necessary equilibrium condition is that lenders' expected profits are the same across contracts offered and demanded with positive probability. Moreover, all entrepreneurs must have access to all contracts. Also, since financial resources are abundant, at least some of the lenders must have a probability lower than one to matching with an entrepreneur (in a symmetric equilibrium, all lenders face the same probability lower than one to match with an entrepreneur). That given, we now show that in both cases, any of these lenders has a strictly profitable deviation. Let us examine case a, first. Consider a contract, $\mathcal{C}' = \{R^{B'}, C', \pi'\}$, with $\mathcal{C}' \neq \mathcal{C}_p$, such that $R^{B'} = R_p^B - \epsilon$, with $\epsilon > 0$, $C' = C_p$, and $\pi' = \pi_p$. Clearly, $\mathcal{C}'$ would attract all types of entrepreneurs, who would be strictly better off since the cost of credit is lower than what they would pay in equilibrium, while the implied level of guarantees stays the same. Therefore, if a lender who in equilibrium has a probability lower than one of matching with an entrepreneur deviates and offers $\mathcal{C}'$, the expected quality of perspective applicants would be qualitatively the same as the one of entrepreneurs' demanding the equilibrium contract, $\mathcal{C}_p$. Therefore, for ϵ sufficiently small, $\mathcal{C}'$ is a strictly profitable deviation as it guarantees the lender a higher expected payoff compared to $\mathcal{C}_p$, as the lender would match with an entrepreneur with probability one. Consider now case b. In this case, the strictly profitable deviation would be to offer a contract $\mathcal{C}'_L \neq \mathcal{C}_L$, with $R_L^{B'} = R_L^B - \epsilon$, with $\epsilon > 0$, $C'_L = C_L$, and $\pi'_L = \pi_L$. Such deviation attracts type-L entrepreneurs might even attract type-H ones, and it is surely profitable for ϵ sufficiently small, as the lender would match with an entrepreneur with probability one. Therefore, in any equilibrium, lenders must be making zero profits. $\square$

According to the above lemma, in equilibrium, given a pair of separating contracts demanded with positive probability, lenders' PCs are satisfied as strict equality, such that:

$$p_H R_H^B + (1-p_H)\beta G_H = 1+r \Rightarrow R_H^B = \frac{(1+r)}{p_H} - \frac{(1-p_H)\beta G_H}{p_H}, \tag{A.1}$$

$$p_L R_L^B + (1-p_L)\beta G_L = 1+r \Rightarrow R_L^B = \frac{1+r}{p_L} - \frac{(1-p_L)\beta G_L}{p_L}. \tag{A.2}$$

ii. Guarantees, collateral and access to credit for type-L borrowers. Solving equation (A.2) for R_L^B and substituting in equation (2) we find the equilibrium value of the payoff of a type-L entrepreneur applying for a separating contract,

$$u_L = \pi_L[p_L R - (1+r) - (1-p_L)(1-\beta)G_L] + w. \tag{A.3}$$

It is immediate to verify that type-L entrepreneurs' equilibrium payoff decreases in the level of guarantees, G_L. Given $\beta < 1$, asset liquidation in case of default is a more expensive way for entrepreneurs to pay back lenders than paying R_L^B in case of success. Consequently, any separating contract designed for type-L entrepreneurs, $\mathcal{C}_L = \left\{R_L^B, C_L, \pi_L\right\}$, demanded with positive probability in equilibrium, must be characterized by minimum guarantees, i.e., $C_L \leq \max\{w - \eta, 0\}$, so that, $G_L = \max\{w - \eta, 0\}$. Consider, by contradiction, a candidate equilibrium such that the level of guarantees, G_L, associated with the equilibrium contract, $\mathcal{C}_L$, satisfies $G_L > \max\{w - \eta, 0\}$. We prove that there always exists a contract $\mathcal{C}_L^{'} \neq \mathcal{C}_L$, which is a strictly profitable deviation. Consider a contract $\mathcal{C}_L^{\prime} = \left\{R_L^{B'}, G_L^{'}, \pi_L^{'}\right\}$, characterized by $G_L^{'} < G_L$, $\pi_L^{'} = \pi_L$, and $R_L^{B'} > R_L^B$

$$p_L R_L^{B'} + (1 - p_L)\beta G_L^{'} = 1 + r + \epsilon. \tag{A.4}$$

Such a contract makes lenders strictly better off for any $\epsilon > 0$, if demanded by at least type-L entrepreneurs. The payoff of an entrepreneur of type-L applying for such contract would be

$$u_L^{'} = \pi_L[p_L R - (1 + r + \epsilon) - (1 - p_L)(1 - \beta)G_L^{'}] + w. \tag{A.5}$$

where we use, $\pi_L = \pi_L^{'}$. Clearly, since the equilibrium payoff of type-L entrepreneurs is strictly decreasing in the level of guarantees (see equation (A.3)), $G_L^{'} < G_L$ implies that type-L are strictly better off applying for $\mathcal{C}_L^{'}$ rather than for the candidate equilibrium contract $\mathcal{C}_L$, for $\epsilon \to 0^+$. So, in equilibrium, no separating contract demanded with positive probability exists with $G_L > \max\{w - \eta, 0\}$. Accordingly, in any equilibrium separating contract, $\mathcal{C}_L$ is characterized by collateral, $C_L \leq \max\{w - \eta, 0\}$. Regarding the probability of access to credit, consider a candidate equilibrium in which the probability of access to credit associated with some separating contract $\mathcal{C}_L = \{R_L^B, C_L, \pi_L\}$ designed for type-L entrepreneurs and demanded with positive probability satisfies $\pi_L < 1$. Lenders have a strictly profitable deviation, which is to offer $\mathcal{C}_L^{'} = \{R_L^B + \epsilon, C_L, 1\}$, where we note that such a deviation will surely attract at least entrepreneurs of type-L as long as $\epsilon > 0$ is sufficiently small. Hence, in any equilibrium, $\pi_L = 1$ for all separating contracts $\mathcal{C}_L$ designed for type-L entrepreneurs and demanded with positive probability. The expected result of no distortion at the bottom applies.

iii. Guarantees, collateral and access to credit for type-H borrowers. Given the values R_H^B, R_L^B, and π_L associated with equilibrium separating contracts demanded with positive probability as derived above, the corresponding values for π_H and G_H associated with the equilibrium separating contract, $\mathcal{C}_H$ are found by solving the following problem:[24]

$$\max_{\{\pi_H, G_H\}} \quad \pi_H \left[p_H R - (1+r) - (1-p_H)(1-\beta)G_H\right] + w \tag{A.6}$$

s.to

$$\pi_H \left[p_H R - (1+r) - (1-p_H)(1-\beta)G_H\right] - \left\{p_H \left(R - \frac{1+r}{p_L}\right) - G_L \left[1 - p_H - \frac{p_H}{p_L}(1-p_L)\beta\right]\right\} \geq 0, \tag{A.7}$$

$$\left[p_L R - (1+r) - (1-p_L)(1-\beta)G_L\right] - \pi_H \left[p_L(R - \frac{1+r}{p_H}) - G_H \left[(1-p_L) - \frac{p_L}{p_H}\beta(1-p_H)\right]\right] \geq 0, \tag{A.8}$$

$$\pi_H \geq 0, \tag{A.9}$$

$$1 - \pi_H \geq 0, \tag{A.10}$$

$$G_H \geq \max\{w - \eta, 0\} \tag{A.11}$$

$$w - G_H \geq 0. \tag{A.12}$$

Note that we choose to characterize the value of the guarantees, G_H, implied by the equilibrium separating contract for type-H entrepreneurs, which is sufficient to characterize the associated level of collateral, C_H, using equation (1) and condition (7). Based on the lagrangean, $\mathcal{L}$, associated with the above problem, the FOCs are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \pi_H} &= (1+\lambda_{ICC,H})(p_H R - (1+r) - (1-p_H)(1-\beta)G_H) - \overline{\lambda}_\pi + \underline{\lambda}_\pi \\
&- \lambda_{ICC,L}\left[p_L(R - \frac{1+r}{p_H}) - G_H\left[(1-p_L) - \frac{p_L}{p_H}\beta(1-p_H)\right]\right] = 0, && \text{(A.13)} \\
\frac{\partial \mathcal{L}}{\partial G_H} &= -\pi_H(1+\lambda_{ICC,H})(1-p_H)(1-\beta) + \underline{\lambda}_G - \overline{\lambda}_G \\
&+ \lambda_{ICC,L}\pi_H\left[1 - p_L - \frac{p_L}{p_H}(1-p_H)\beta\right] = 0. && \text{(A.14)}
\end{aligned}$$

where we denote with λ the lagrangean multipliers. The $\lambda_{ICC,\theta}$, with $\theta = L, H$ are the multipliers associated with the ICC constraints for type-L and type-H entrepreneurs. $\overline{\lambda}_\pi$ and $\underline{\lambda}_\pi$ are associated with the constraints, (A.10) and (A.9), respectively, while $\underline{\lambda}_G$ and $\overline{\lambda}_G$ are associated with constraints, (A.11) and (A.12).

Case 1: Wealth constraints are not binding, i.e., $G_H \in (w_\eta, w)$**.** In this case, $\underline{\lambda}_G = \overline{\lambda}_G = 0$. Imposing this restriction, condition (A.14) can be rewritten as

$$\pi_H(1+\lambda_{ICC,H})(1-p_H)(1-\beta) = \lambda_{ICC,L}\pi_H\left[1 - p_L - \frac{p_L}{p_H}(1-p_H)\beta\right]. \tag{A.15}$$

[24] Note that we use (A.1) and (A.2) to substitute for the equilibrium values of R_H^B and R_L^B as functions of G_H in the objective function and in the constraints.

It is immediate to verify that, given assumption 1, equation (7), for any level of wealth $w \in [0, \overline{w}]$, the expected equilibrium payoff entrepreneurs of type-H financed by means of a separating contract

$$u_H = p_H R - (1+r) - (1-p_H)(1-\beta)G_H + w > 0® \tag{A.16}$$

is strictly greater than w, for any feasible value of guarantees, G_H. Therefore, type-H entrepreneurs are strictly better off the higher the probability of having access to credit is. Accordingly, $\pi_H = 1$ should hold if a borrower is not wealth-constrained. Based on this result, we solve the maximization problem imposing, $\pi_H = 1$ as part of the solution. Given $\pi_H = 1$, equation (A.15) implies $\lambda_{ICC,L} > 0$, so that condition (A.8) holds as a strict equality. Imposing strict equality, substituting $\pi_H = 1$, and solving (A.8) for G_H yields

$$G_H = \frac{(1+r)(p_H - p_L) + p_H(1-p_L)(1-\beta)G_L}{(1-p_L)p_H - p_L(1-p_H)\beta}. \tag{A.17}$$

Note that $G_H > G_L$ for $G_L \leq \frac{1+r}{\beta}$ holds. The implied level of collateral, C_H, satisfies $C_H = G_H$.

iii. Credit Market participation by entrepreneurs. Given that any equilibrium separating contract demanded with positive probability guarantees lenders an expected return equal to $1+r$, assumption 1, equation (7), implies that all entrepreneurs in equilibrium, strictly benefit from applying for credit, and therefore they all participate.

iv. Incentive compatibility. Finally, having characterized the equilibrium separating contract for a given level of wealth w, we verify that the ICC_H holds. The following result applies

Lemma 4. *When the ICC_L is binding, the ICC_H is satisfied if and only if the following inequality holds:*

$$\frac{\pi_H}{\pi_L} \geq \frac{R - R_L^B + G_L}{R - R_H^B + G_H}. \tag{A.18}$$

Proof. If the ICC_L is binding, then

$$\pi_L \left[p_L(R - R_L^B) - (1-p_L)G_L \right] = \pi_H \left[p_L(R - R_H^B) - (1-p_L)G_H \right]. \tag{A.19}$$

Adding and subtracting $\pi_H \left[p_H(R - R_H^B) - (1-p_H)G_H \right]$, we obtain

$$\begin{aligned} &\pi_H(p_H - p_L)(R - R_H^B + G_H) + \pi_L \left[p_L(R - R_L^B) - (1-p_L)G_L \right] = \\ &\pi_H \left[p_H(R - R_H^B) - (1-p_H)G_H \right]. \end{aligned} \tag{A.20}$$

Moreover, by adding and subtracting $\pi_L\left[p_H(R-R_L^B)-(1-p_H)G_L\right]$ from the expression for the payoff of an entrepreneur of type-L, we obtain

$$\begin{aligned}&\pi_L\left[p_L(R-R_L^B)-(1-p_L)G_L\right]= \\ &\pi_L\left[p_H(R-R_L^B)-(1-p_H)G_L\right]-\pi_L(R-R_L^B+G_L)(p_H-p_L).\end{aligned} \tag{A.21}$$

Using (A.21) to substitute for $\pi_L\left[p_L(R-R_L^B)-(1-p_L)G_L\right]$ in (A.20), we find

$$\begin{aligned}&\pi_H\left[p_H(R-R_H^B)-(1-p_H)G_H\right]-\pi_L\left[p_H(R-R_L^B)-(1-p_H)G_L\right]= \\ &(p_H-p_L)\left[\pi_H(R-R_H^B+G_H)-\pi_L(R-R_L^B+G_L)\right],\end{aligned} \tag{A.22}$$

where the RHS is positive if and only if

$$\frac{\pi_H}{\pi_L}\geq\frac{R-R_L^B+G_L}{R-R_H^B+G_H}. \tag{A.23}$$

□

In the case we are analyzing, $\pi_H=\pi_L=1$ holds, so that condition (A.23) reduces to

$$R-R_L^B+G_L\leq R-R_H^B+G_H. \tag{A.24}$$

For $G_L<(1+r)/\beta+\eta$, $R_H^B<R_L^B$ holds, so that, given $G_H>G_L$, the above inequality is always satisfied, which proves that the ICC_H holds. The above analysis relies on the assumption that the optimal solution satisfies PC_H. Substituting for the equilibrium contract, the PC_H reduces to

$$p_HR-(1+r)-(1-\beta)(1-p_H)G_H\geq 0\Rightarrow G_H\leq\frac{p_HR-(1+r)}{(1-\beta)(1-p_H)}, \tag{A.25}$$

where we note that the LHS is strictly decreasing in G_H. Given assumption 1, equation (7), $G_H<(1+r)/\beta$. Hence, the PC_H is always satisfied as long as

$$\frac{p_HR-(1+r)}{(1-\beta)(1-p_H)}\geq\frac{1+r}{\beta}, \tag{A.26}$$

which is the equivalent to the parameter restriction of assumption 1, equation (6). Finally, given $G_H<(1+r)/\beta$, $G_L<(1+r)/\beta$ follows, which suffices for the PC of type-L agents to be satisfied.

Case 2: $G_H\in(\max\{w-\eta,0\},w]$ **is binding.** This case applies when

$$\frac{(1+r)(p_H-p_L)+p_H(1-p_L)(1-\beta)G_L}{(1-p_L)p_H-p_L(1-p_H)\beta}>w, \tag{A.27}$$

so that, safe entrepreneurs cannot afford the level of guarantees needed to enter contracts designed for their type and characterised by a probability of access to credit, $\pi_H = 1$. It follows from condition (A.14) that ICC_L must be binding. Imposing strict equality and solving for π_H we find

$$\pi_H = \frac{p_L R - (1+r) - (1-p_L)(1-\beta)w_\eta}{p_L R - \frac{p_L}{p_H}(1+r) - (1-p_L)w(1 - \frac{1-p_H}{1-p_L}\frac{p_L}{p_H}\beta)}. \tag{A.28}$$

We now check that the above solution satisfies the ICC_H. Substituting for the equilibrium values of π_H and π_L, imposing $G_L = \max\{w-\eta, 0\}$ and $G_H = w$, (A.23) (lemma 4) can be rewritten as

$$\frac{R - R_L^B + w_\eta - \frac{w_\eta}{p_L}}{R - R_H^B + w - \frac{w}{p_L}} \geq \frac{R - R_L^B + w_\eta}{R - R_H^B + w}. \tag{A.29}$$

We distinguish two cases: (1) $w \leq \eta$, which implies $\max\{w-\eta, 0\} = 0$; and (2) $w > \eta$, which implies $\max\{w-\eta, 0\} = w - \eta$. In case 1, condition (A.29) reduces to

$$\frac{R - R_L^B}{R - R_H^B + w - \frac{w}{p_L}} \geq \frac{R - R_L^B}{R - R_H^B + w}, \tag{A.30}$$

and thus, ICC_H is always satisfied. In contrast, in case (2), (A.29) reduces to

$$w(R_L^B - R_H^B) \leq \eta(R - R_H^B). \tag{A.31}$$

In this case, whether the ICC_H holds or not is less obvious. However, in the proof of proposition 1, we show that the existence of an equilibrium in which poor entrepreneurs of wealth $w - \eta > 0$ requires that these entrepreneurs prefer the equilibrium separating contract $\mathcal{C}_H$ to any equilibrium pooling contract. This implies that they also prefer, $\mathcal{C}_H$ to a pooling contract characterized by minimum guarantees, $G_p = w - \eta$. We observe that such contract would be always strictly preferred to the contract designed for type-L entrepreneurs, $\mathcal{C}_L$. It then follows that whenever a candidate equilibrium meets the above necessary condition such that poor entrepreneurs of type H with $w - \eta$ prefer $\mathcal{C}_H$ to any $\mathcal{C}_p$ and separate, the ICC_H is satisfied and (A.31) holds. □

A.2 Proof of lemma 2

We provide a complete characterization of the pooling equilibrium contracts for a given w, under the assumption that entrepreneurs decide to disclose their wealth when borrowing. In section A.3, we prove that this is indeed the case. Given a candidate equilibrium in which the pooling contract $\mathcal{C}_p = \{R_p^B, C_p, \pi_p)$ is being offered and demanded with positive probability, lemma 3 applies, so that

$$p_m R_p^B + (1-p_m)\beta G_p = 1 + r, \tag{A.32}$$

where $G_p = \max\{w-\eta, C_p\}$ is the level of guarantees implied by $\mathcal{C}_p$, and where $p_m = \mu p_H + (1-\mu)p_L$. Accordingly, the payoff of an entrepreneur of type-θ, with $\theta = H, L$, and wealth w is

$$u_\theta^{PE} = \pi_p \left\{ Rp_\theta - (1+r)\frac{p_\theta}{p_m} + G_p \left[(1-p_m)\frac{p_\theta}{p_m}\beta - (1-p_\theta) \right] \right\} + w. \tag{A.33}$$

It can be immediately verified that, given assumption 1, $u_\theta^{PE} > 0$, for $\theta = L, H$. Therefore, in any PE, all entrepreneurs demand credit. Moreover, this also implies that given a candidate PE such that $\pi_p < 1$, there always exist profitable deviations, which destroy the candidate equilibrium. Hence, $\pi_p = 1$ must hold. Finally, given expression (A.33), if

$$\beta\frac{(1-p_m)}{p_m} < \frac{1-p_H}{p_H} \tag{A.34}$$

both types of entrepreneur prefer less guarantees, so that $C_p = 0$, and $G_p = \max\{w-\eta, 0\}$ holds. If the reverse inequality holds, then safe entrepreneurs prefer more guarantees, $C_p = w$, and hence, any pooling contract must be characterised by $G_p = \min\left\{w, \frac{1+r}{\beta}\right\}$. $\square$

A.3 Incentives to disclose wealth

Lemma 5 (Wealth disclosure)**.** *In any equilibrium, entrepreneurs disclose their wealth ex ante.*

Proof. We first analyse the decision of type-H entrepreneurs to disclose their wealth if applying for a separating contract. In equilibrium, any separating contract designed for type-H entrepreneurs, $\mathcal{C}_H$ and demanded with positive probability satisfy the ICC_L as a strict equality:

$$[p_L(R-(1+r)-(1-\beta)(1-p_L)G_L] = \pi_H[p_L(R-R_B^H)-(1-p_L)G_H]. \tag{A.35}$$

The LHS of the above constraint is decreasing in G_L. Let $\mathcal{E}$ be the set of entrepreneurs of type-H applying for $\mathcal{C}_H$, who are not disclosing their wealth, and $\overline{w}(\mathcal{E})$ the highest value of individual wealth of entrepreneurs in that set. Since G_L is increasing in w, the level of guarantees, G_H, implied by the contract $\mathcal{C}_H$ demanded by any entrepreneur of type-H in $\mathcal{E}$ must satisfy the following ICC_L,

$$[p_L R - (1-p_L)(1-\beta)\max\{\overline{w}(\mathcal{E})-\eta, 0\}] = \pi_H[p_L(R-R_H^B)-(1-p_L)G_H]. \tag{A.36}$$

Crucially, for a type-L entrepreneur with a wealth $w' < \overline{w}(\mathcal{E})$, the above ICC_L would be satisfied as a strict inequality. Hence, entrepreneurs of type-H endowed with wealth, w', have an incentive to disclose their wealth because in that case they can be offered a separating contract, $\mathcal{C}_H^{'}$, which satisfies as strict equality the ICC_L for type-L entrepreneurs endowed with wealth w',

$$\pi_L[p_L(R-R_L^B)-(1-p_L)\max\{w'-\eta, 0\}] = \pi_H[p_L(R-R_H^B)-(1-p_L)G_H]. \tag{A.37}$$

Clearly, such contract, $C_H^{'}$, allows either for a greater probability of having access to credit (for poor and safe entrepreneurs), or a lower level of guarantees (for rich and safe entrepreneurs) compared to the contract $\mathcal{C}_H$, or both. This directly implies that, given a candidate equilibrium in which some type-H entrepreneurs are not disclosing their wealth, lenders have an incentive to propose separating contracts such as $C_H^{'}$ that require wealth disclosure, as by doing so they can surely attract borrowers and make extra profits. Let us now turn to decision of entrepreneurs of type-L, who are self-selecting in a separating contract C_L (on offer). Let $\mathcal{E}'$ the set of type-L entrepreneurs demanding such contract who are not disclosing their wealth. Define, $E(w|e \in \mathcal{E}^{'})$, the conditional expected value of wealth associated to the set $\mathcal{E}^{'}$ of entrepreneurs. In equilibrium, lenders break even in expected terms, given the information available. Hence, for each entrepreneur of type-L not disclosing her wealth, the equilibrium contract satisfies

$$p_H R_L^B + (1 - p_H)\beta E(w|e \in \mathcal{E}^{'}) = 1 + r. \tag{A.38}$$

It is then immediate to verify that if disclosing her wealth, the richest entrepreneur who is not disclosing it would be better off, as lenders would find it profitable to offer her a new contract with a lower cost of credit. Equivalent arguments hold for the case of a pooling contract. □

A.4 Proof of proposition 1

In order to characterized the equilibrium for any given level of wealth, w, and provide the necessary and sufficient conditions for its existence, we study whether given a candidate equilibrium that involves either pooling or separation for a given value of wealth, $w \in [0, \overline{w}]$, there exist strictly profitable deviations available to lenders. We focus first on contracts designed for rich entrepreneurs, i.e., entrepreneurs with $w \geq \widehat{w}_1$ and then we turn to poor ones, with $w < \widehat{w}_1$.

Case a: Rich entrepreneurs. Under the unique candidate equilibrium, the payoff of a rich type-H entrepreneur who self-selects in the separating contract, $\mathcal{C}_H$, characterized in lemma 1, is

$$p_H R - (1 + r) - (1 - \beta)(1 - p_H)G_H. \tag{A.39}$$

Clearly, under pure strategies, the best alternative contract that lenders could offer to entrepreneurs of type-H is the pooling contract, $\mathcal{C}_p$, characterized in lemma 2. If an entrepreneur of type-H could apply for, $\mathcal{C}_p$, her payoff would be

$$p_H R - (1 + r)\frac{p_H}{p_m} - G(1 - p_H) + \beta(1 - p_m)\frac{p_H}{p_m}G_p. \tag{A.40}$$

Hence, given a candidate equilibrium in which one or more rich entrepreneurs of type H are self-selecting into separating contracts, a strictly profitable deviation for lenders exists if and only if

$$\underbrace{p_H R - (1+r) - (1-\beta)(1-p_H)G_H}_{separating} < \underbrace{p_H R - (1+r)\frac{p_H}{p_m} + G_p(1-p_H)[\beta\frac{1-p_m}{1-p_H}\frac{p_H}{p_m} - 1]}_{pooling}, \quad \text{(A.41)}$$

which rearranging reduces to

$$(1+r)\frac{p_H - p_m}{p_m} < (1-\beta)(1-p_H)G_H - p_H G_p\left[\frac{1-p_H}{p_H} - \beta\frac{1-p_m}{p_m}\right], \quad \text{(A.42)}$$

where

$$G_H = \frac{(1+r)(p_H - p_L) + p_H(1-p_L)(1-\beta)G_L}{(1-p_L)p_H - p_L(1-p_H)\beta}. \quad \text{(A.43)}$$

Consider now an equilibrium in which some rich entrepreneurs select the pooling contract, $\mathcal{C}_p$. Their payoff would be given by equation (A.40). The best deviation lenders could conceive with respect to these entrepreneurs would be to offer the menu of equilibrium separating contracts $\{\mathcal{C}_H, \mathcal{C}_L\}$, defined by lemma 1. To see why, note that in the subgame played following a deviation, if entrepreneurs of type-H strictly prefer the off equilibrium contract on offer, the optimal strategy of lenders offering the equilibrium contract would be to refuse applicants, as the only ones to apply for the pooling contract would be type L entrepreneurs, which implies that the pooling contract would always result in a loss for lenders accepting applicants. Accordingly, the optimal strategy of type-L entrepreneurs in the subgame following a deviation would be to apply for the off equilibrium contracts too, so long as the off equilibrium contract is strictly preferred by type-H entrepreneurs. Accordingly, the only profitable deviation, if any, is to offer a menu of separating contracts.[25] Such deviation would be strictly profitable if and only if

$$\underbrace{p_H R - (1+r) - (1-\beta)(1-p_H)G_H}_{separating} > \underbrace{p_H R - (1+r)\frac{p_H}{p_m} + G_p(1-p_H)[\beta\frac{1-p_m}{1-p_H}\frac{p_H}{p_m} - 1]}_{pooling}, \quad \text{(A.44)}$$

which is the reverse of (A.41). The following preliminary result holds

Lemma 6. *If the fraction of type-H entrepreneurs in the population, μ, satisfies*

$$\mu < \mu_2 \equiv \frac{p_H\beta(1-p_L) - (1-p_H)p_L}{[\beta p_H + (1-p_H)x]\,(p_H - p_L)} \quad \text{(A.45)}$$

[25] Note that not allowing for lenders to accept/refuse applicants would imply that the optimal deviation would be to offer a contract that only type-H find profitable given that type-L still have the possibility to access the pooling contract. Under this game structure, the existence of a pure strategy equilibrium is not guaranteed as discussed by Hellwig (1987).

adverse selection is so severe that, given a candidate equilibrium in which rich entrepreneurs of type-H separate, there is no strictly profitable deviation for lenders, while given a candidate equilibrium in which these entrepreneurs are pooled with type-L ones a strictly profitable deviation for lenders exists.

Proof. It is immediate to verify that if μ satisfies (A.45), then from lemma 2, the equilibrium pooling contract, $\mathcal{C}_p$, implies a level of guarantees, $G_p = w$. We know that for rich and safe entrepreneurs, the separating equilibrium contract, $\mathcal{C}_H$, implies $G_H \leq w$, as these entrepreneurs are not wealth constrained when applying for such contract. Therefore, given a candidate equilibrium in which rich entrepreneurs of type-H separate, $\mathcal{C}_p$, which is the best contract lenders might offer if deviating, implies a level of guarantees higher than that rich and safe entrepreneurs face under $\mathcal{C}_H$, i.e $G_p \geq G_H$. Safe and rich entrepreneurs would prefer a separating contract with guarantees G_p, if available, than $\mathcal{C}_p$, as they would pay a lower interest rate.[26] In turns, such contract would be always dominated by $\mathcal{C}_H$, which is characterized by lower guarantees. Therefore, $\mathcal{C}_p$ is not a profitable deviation as it never attracts safe entrepreneurs if (A.45) holds. This constitutes a necessary and sufficient condition for the existence of an equilibrium such that rich entrepreneurs of type-H separate as well as for the non existence of an equilibrium in which they pool with risky ones.□

Given the above lemma, we know that if $\mu \leq \mu_2$, separation takes place among rich entrepreneurs, i.e,. entrepreneurs with wealth, $w \geq \widehat{w}_1$. To further explore the existence of an equilibrium that involves separation/pooling for levels of wealth $w \geq \widehat{w}_1$, we turn our attention to the case in which $\mu \geq \mu_2$. We have to consider two sub-cases. Sub-case 1, in which $\widehat{w}_1 \geq \eta$, so that $w > \eta$ holds for any rich entrepreneur, and sub-case 2, in which $\widehat{w}_1 < \eta$, so that $w < \eta$ holds for some rich entrepreneurs. It is important to note that, in both cases, given $\mu \geq \mu_2$, the level of guarantees associated with the equilibrium pooling contract, $\mathcal{C}_p$, would be $G_p = \max\{w - \eta, 0\}$.

Sub-case 1: $w \in (\eta, \frac{1+r}{\beta} + \eta)$ **for all rich entrepreneurs**. The levels of guarantees associated with equilibrium separating contracts, $\{\mathcal{C}_H, \mathcal{C}_L\}$, designed for type-$L$ and type-H entrepreneurs are

$$G_L = w - \eta \tag{A.46}$$

$$G_H = \frac{(1+r)(p_H - p_L) + p_H(1-p_L)(1-\beta)(w-\eta)}{(1-p_L)p_H - p_L(1-p_H)\beta} \tag{A.47}$$

Using the above values to substitute into equation (A.41), we find that a strictly profitable deviation

[26] $\mathcal{C}_p$ guarantees separation as it involves a level of guarantees higher than the minimum needed to separate, G_H.

exists, which consists in offering the contract $\mathcal{C}_p$, if and only if

$$\mu > \frac{(p_H - p_L)}{(1-\beta)p_H(1-p_H) + (p_H - p_L)} \equiv \mu_1. \tag{A.48}$$

Accordingly, an equilibrium in which rich entrepreneurs separate exists if and only if $\mu \leq \mu_1$. Using (A.46) and (A.47) to substitute into condition (A.44) it is immediate to verify that an equilibrium in which rich entrepreneurs pool exists if and only if, $\mu \geq \mu_1$.

Sub-case 2: $w < \eta$ for some rich entrepreneurs. For rich entrepreneurs with $w > \eta$ the above analysis hold. For rich entrepreneurs with $w < \eta$, in equilibrium, separating contracts imply

$$G_L = 0 \tag{A.49}$$

$$G_H = \frac{(1+r)(p_H - p_L)}{(1-p_L)p_H - (1-p_H)p_L\beta} \tag{A.50}$$

By substituting equations (A.49) and (A.50) into equation (A.42), we find that also in this sub-case, the necessary and sufficient condition for the existence of an equilibrium in which all rich entrepreneurs separate is $\mu \leq \mu_1$. The same logic as before applies to the existence of an equilibrium in which such entrepreneurs pool, so that the related necessary and sufficient condition is $\mu \geq \mu_1$.

Finally, it is immediate to verify that $\mu_1 > \mu_2$. Accordingly, combining the analysis of sub-cases 1 and 2 with lemma 6 leads to the conclusion that for entrepreneurs with wealth $w \geq \widehat{w}_1$, the equilibrium involves separation if $\mu < \mu_1$ and pooling otherwise.

Case b: Poor entrepreneurs. The equilibrium payoff of a poor and safe entrepreneur who separates is

$$\pi_H[p_H R - (1+r) - (1-p_H)(1-\beta)G_H]. \tag{A.51}$$

Also in this case, the best alternative contracts that could be offered to entrepreneurs who are separating is $\mathcal{C}_p$ (the same argument as in case a above holds). If the entrepreneur could apply for $\mathcal{C}_p$, the payoff would be

$$p_H R - (1+r)\frac{p_H}{p_m} - (1-p_H)G_p\left[1 - \frac{1-p_m}{p_m}\frac{p_H}{1-p_H}\beta\right]. \tag{A.52}$$

Therefore, there exists a profitable deviation for lenders if and only if

$$\pi_H[p_H R - (1+r) - (1-p_H)(1-\beta)G_H] < p_H R - (1+r)\frac{p_H}{p_m} - p_H G_p\left[\frac{1-p_H}{p_H} - \beta\frac{1-p_m}{p_m}\right]. \tag{A.53}$$

The reverse necessary and sufficient condition holds for the existence of strictly profitable deviations, given an equilibrium in which (some) poor entrepreneurs are pooling. We consider first the case in which $\mu \geq \mu_2$, so that according to lemma 2, $\mathcal{C}_p$ is characterized by minimum guarantees, i.e

$G_p = \max\{w-\eta, 0\}$. Both the RHS and the LHS are continuous and differentiable in w. The LHS of condition (A.53) is strictly increasing in w. As for the RHS, we note that $\mu \geq \mu_2$ implies

$$\beta \frac{1-p_m}{p_m} < \frac{1-p_H}{p_H}. \tag{A.54}$$

Accordingly, the RHS of (A.53) is strictly decreasing in w. Moreover, for $w \to 0$ (A.53) reduces to

$$-\left(p_H R - (1+r)\right)\left(1 - \frac{p_L}{p_m}\right) < 0. \tag{A.55}$$

Thus, continuity implies that for sufficiently low levels of wealth, poor and safe entrepreneurs prefer pooling. We know from the analysis of Case a above, that for $w \to \widehat{w}_1$, separation occurs if $\mu \leq \mu_1$, and pooling prevails otherwise. Then, by continuity, if $\mu \leq \mu_1$ there exist a value of $w < \widehat{w}_1$, call it $\widehat{w}_2$ such that in equilibrium, poor entrepreneurs with $w \leq \widehat{w}_2$ pool, while those with $w \geq \widehat{w}_2$ separate. Viceversa, if $\mu \geq \mu_1$, in any equilibrium all poor entrepreneurs pool. Let us now turn to the case in which $\mu \leq \mu_2$. We know that $\mu_1 > \mu_2$. Hence, $\mu \leq \mu_2$ implies $\mu < \mu_1$ so that poor and safe entrepreneurs with $w \to \widehat{w}_1$ prefer to separate; i.e., (A.53) is violated for $w \to \widehat{w}_1$. Furthermore, as in the previous case, condition (A.53) is verified for $w \to 0$ so that any equilibrium involves pooling for sufficiently poor entrepreneurs. Finally, in this case, the RHS of (A.53) is linear and increasing in w. The LHS is strictly increasing and convex in w.[27] This, given the other properties discussed above, ensures that if $\mu \leq \mu_1$ there exists a unique level of wealth $\widehat{w}_2 \in (0, \widehat{w}_1]$ such that in any equilibrium poor and safe entrepreneurs with $w \geq \widehat{w}_2$ separate, while those with $w < \widehat{w}_2$ pool. Viceversa, if $\mu > \mu_1$ a all poor entrepreneurs pool. $\square$

[27] The first derivative of the LHS of (A.53) with respect to w is

$$\overline{\lambda}_G = \pi_H \left[\lambda_{ICC,L}\left(1 - p_L - \frac{p_L}{p_H}(1-p_H)\beta\right) - (1-p_H)(1-\beta)\right] \tag{A.56}$$

which is strictly positive as $(1-p_H)(1-\beta) < 1 - p_L - p_L(1-p_H)\beta/p_H$, and

$$\lambda_{ICC,L} = \frac{p_H(R-R_H) - (1-p_H)G_H}{p_L(R-R_H) - (1-p_L)G_H} \tag{A.57}$$

which is greater than 1, which ensures that the expression in brackets is positive. The second order derivative is

$$\frac{\partial \overline{\lambda}_G}{\partial w} = \frac{\partial \pi_H}{\partial w}\left[\lambda_{ICC,L}(1 - p_L - \frac{p_L}{p_H}(1-p_H)\beta) - (1-p_H)(1-\beta)\right] + \pi_H\left(1 - p_L - \frac{p_L}{p_H}(1-p_H)\beta\right)\frac{\partial \lambda_{ICC,L}}{\partial w} > 0. \tag{A.58}$$

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Table 1: Loan rate and fraction of rationed firms by exemption, collateral and assets

		Any asset			Low asset			High asset		
		FS	LEX	HEX	FS	LEX	HEX	FS	LEX	HEX
C=1	Loan rate (%)	5.49	5.53	5.37	6.06	6.13	5.89	5.08	5.12	4.96
C=0	Loan rate (%)	6.19	6.06	6.57	6.95	6.8	7.42	5.04	5.02	5.12
C=1	Rationed firms (%)	4.5	4.7	3.8	8.1	8.6	6.7	1.9	2.0	1.5
C=0	Rationed firms (%)	3.0	1.9	6.5	3.7	2.3	7.7	1.9	1.2	4.4
Any firm	Loan rate (%)	5.81	5.78	5.90	6.55	6.50	6.70	5.06	5.08	5.02
Any firm	Rationed firms (%)	3.8	3.4	5.0	5.7	5.1	7.2	1.9	1.7	2.5

FS: Full sample; LEX: Low exemption; HEX: High exemption. Low asset: assets below median value; High asset: assets above the median value.

Table 2: Probability to post collateral

	Coefficient
Dummy=1 if firm's credit score is in the top 25%	-0.9022***
Loan original maturity (n. of months)	0.0050***
Amount granted over total applied	-0.1786***
Banking market concentration: Dummy=1 if Herfindahl index>1800	0.1216***
Dummy=1 if firm has limited liability	0.3031***
Dummy=1 if owner is female	-0.2112***
Years of firm-bank relationship	-0.0042***
Dummy=1 if firm is family owned	-0.0837***
N	1615
χ^2	105.6

Significance levels: $*$: 10% $**$: 5% $***$: 1%.

Table 3: Probability of having access to credit - marginal effects

	(1)	**(2)**
Dummy=1 if firm located in high-exemption area	-0.0289***	-0.0176***
Dummy=1 if firm posted collateral	-0.0113***	-0.0114**
Dummy=1 if firm posts collateral and is located in high-exemption area	0.0128***	0.0218**
$R_L^B/R_H^{\prime B}$	-1.0294***	-1.0242***
Loan original maturity (n. of months)	-0.0002***	-0.0002***
Amount granted over total applied	0.0445***	0.0448***
Years of firm-bank relationship	0.0005***	0.0004**
Dummy=1 if firm's credit score is top 25%	0.0108***	0.0117**
Dummy=1 if firm has delinquency records	-0.0051***	-0.0051 ***
Debts over equity	-0.0001**	-0.0001
Dummy=1 if firm has limited liability	-0.0034	0.0037
Total assets - thousands of $	0.000001***	0.000001***
N	1591	1591
Log-likelihood	-209.42	—
$\chi^2_{(12)}$	86.08	—

Significance levels: $*$: 10% $**$: 5% $***$: 1%. Column (1) reports probit estimation; column (2) probit estimation taking into account the imputation of data.

Table 4: Cost of credit - Switching regression

	$\mathbf{R_L^B}$	$\mathbf{R_H^B}$	$\mathbf{R_H'^B}$	$\mathbf{R_L'^B}$
Inverse Mills ratio λ_i	-0.5447**	-0.2754*	-0.1933	-0.2585**
Dummy=1 if firm located in high-exemption area	0.2178**	-0.2023***	0.2139**	-0.1982***
Total assets - thousands of $	-0.000001	-0.000005**	-0.0000004**	-0.000005*
Total assets× High exemption dummy	-0.00001**	0.0000004	-0.00001**	0.0000004
Dummy=1 if firm's credit score is in the top 25%	-0.1574	-0.0667	-0.1349	-0.0680
Dummy=1 if the fixed interest rate	0.9891***	1.1680***	0.9863***	1.1657***
Dummy=1 if the loan was a new line of credit	-0.1388	-0.4242***	-0.1203	-0.4294***
Banking market concentration: Dummy=1 if Herfindahl index>1800	0.3831***	0.1156**	0.3461***	0.1182**
Owner's managerial experience (n. of years)	-0.0297***	-0.0095***	-0.0300***	-0.0095***
Dummy=1 if owner is black	1.6548***	-0.7073***	1.6626***	-0.7103**
Dummy=1 if owner belongs to an ethnic minority other than black	1.3748***	0.0665	1.3506***	0.0696
Dummy=1 if owner is female	-0.2358*	-0.1359	-0.1876	-0.1355
Dummy=1 if firm is family owned	-0.0939	-0.2815***	-0.0748	-0.2809***
Number of credit applications	-0.0065	0.0434***	-0.0148	0.0438***
Years of firm-bank relationship	-0.0200***	0.0018	-0.0188***	-0.0018
Distance of firm from bank (miles)	0.0014**	-0.0001	0.0015**	-0.0001
Natural log of total sales	-0.3829***	-0.2198***	-0.3844***	-0.2225***
Debt over total asset	0.0237	0.0313***	0.0233	0.0317***
Intercept	12.1981***	8.5600***	11.6574***	9.0331***
N	697	881	697	881
R^2	0.20	0.19	0.20	0.19
F	9.66	12.43	9.61	12.46

Significance levels: $*$: 10% $**$: 5% $***$: 1%.

Table 5: Cost of credit

	(1)	(2)
Dummy=1 if firm located in high-exemption area	0.2612***	0.2354
Dummy=1 if firm posted collateral	-0.3353***	-0.3388**
Dummy=1 if firm posts collateral and is located in high-exemption area	-0.5248***	-0.4958**
Dummy=1 if firm's credit score is top 25%	-0.0936*	-0.0835
Dummy=1 if the fixed interest rate	1.0964***	1.0878***
Dummy=1 if loan was a new line of credit	-0.2311**	-0.2363
Banking market concentration: Dummy=1 if Herfindahl index> 1800	0.2544***	0.2532**
Owner's managerial experience (n. of years)	-0.0164***	-0.0163***
Dummy=1 if owner is black	0.7574***	0.7453
Dummy=1 if owner belongs to an ethnic minority other than black	0.8217***	0.8255***
Dummy=1 if owner is female	-0.0988	-0.0988
Dummy=1 if firm is family owned	-0.2275***	-0.2293*
Number of credit applications	0.0292	0.0295
Years of firm-bank relationship	-0.0091***	-0.0091
Distance of firm from bank (miles)	0.0011***	0.0011
Natural log of total sales	-0.3177***	-0.3199***
Debt over total assets	0.0232***	0.0229
Total assets - thousands of $	-0.000005**	0.000005
Intercept	10.6677***	10.7019***
N	1671	1671
R^2	0.19	-
F	23.23	21.25

Significance levels: $*$: 10% $**$: 5% $***$: 1%. Column (1) reports the OLS estimation; column (2) reports the OLS estimation taking into account the imputation of data.

Table 6: Simultaneous model

	Whole sample	**Low exemption**	**High exemption**
		Dependent variable: R^B	
Guarantees	-0.3467***	-0.1766***	-0.7525***
Dummy=1 if firm's credit score is in the top 25%	-0.1329**	0.0148	-0.7058***
Dummy=1 if the fixed interest rate	1.0631***	1.1377***	0.8918***
Dummy=1 if loan was a new line of credit	-0.3332***	-0.2134	-0.7078***
Banking market concentration:			
Dummy=1 if Herfindahl index> 1800	0.2512***	0.1118*	0.6743***
Owner managerial experience (n. of years)	-0.0157***	-0.0201***	0.0018
Dummy=1 if owner is black	0.6584 ***	0.6765***	0.9328**
Dummy=1 if owner belongs to an ethnic minority other than black	0.6247***	0.2921**	1.3701***
Dummy=1 if owner is female	-0.1081	-0.1634*	0.2247
Dummy=1 if firm is family owned	-0.2239***	-0.2460***	-0.3276**
Number of credit applications	0.0306	0.0280**	-0.0330
Years of firm-bank relationship	-0.0102***	-0.0070**	-0.0201***
Distance of firm from bank (miles)	0.0006***	0.0009**	-0.0006
Natural log of total sales	-0.2725***	-0.2568***	-0.3182***
Debt over total assets	0.0276***	0.0256*	0.0242***
Total assets - thousands of $	-0.000004**	-0.000005*	-0.000003
Intercept	9.8520***	9.6905***	10.4263***
R^2	0.18	0.17	0.25
F	22.65	13.81	8.28
		Dependent variable: G	
Loan Rate	-0.2305***	-0.2420***	-0.1805***
Loan original maturity (n. of months)	-0.0043***	0.0044***	-0.0044***
Amount granted over total applied	-0.1737***	-0.1695***	-0.1699**
Years of firm bank relationship	-0.0072***	-0.0087***	-0.0023
Dummy=1 if firm's Credit score is top 25%	-0.1129***	-0.0533	-0.2983***
Banking market concentration:			
Dummy=1 if Herfindahl index> 1800	0.1486***	0.1520***	0.1221**
Dummy=1 if firm has limited liability	0.0926**	0.0805*	0.1740*
Dummy=1 if owner is female	-0.1539***	-0.2405***	0.0786
Dummy=1 if firm is family owned	-0.1141***	-0.0697	-0.3052***
Intercept	1.5363***	1.5461***	1.2901***
LR χ^2	127.25	115.53	37.73
N	1578	1183	395

Significance levels: $*$: 10% $**$: 5% $***$: 1%.
Two-stage probit least squares estimation (Maddala and Lee, 1976; Keshk, 2003)

Table 7: Cost of credit with selection

	(1)	(2)
Dummy=1 if firm located in high-exemption area	0.2348***	0 .2101
Dummy=1 if firm posted collateral	-0.3484***	-0.3510**
Dummy=1 if firm posts collateral and is located in high-exemption area	-0.4982***	-0.4699*
Dummy=1 if firm's credit score is in the top 25%	0.1002*	0.1152
Dummy=1 if the fixed interest rate	1.0869***	01.0795***
Dummy=1 if loan was a new line of credit	-0.2268***	-0.2309
Banking market concentration: Dummy=1 if Herfindahl index> 1800	0.2531***	0.2531**
Owner's managerial experience (n. of years)	-0.0147***	-0.0146**
Dummy=1 if owner is black	0.7539***	0.7397
Dummy=1 if owner belongs to an ethnic minority other than black	0.8397***	0.8441***
Dummy=1 if owner is female	-0.1208*	-0.1201
Dummy=1 if firm is family owned	-0.3162***	-0.3204**
Number of credit applications	0.0266	0.0267
Years of firm-bank relationship	-0.0092***	-0.0092
Distance of firm from bank (miles)	0.0012***	0.0012
Natural log of total sales	-0.2823***	-0.2832***
Debt over total assets	0.0106	0.0100
Total assets - thousands of $	-0.000005**	0.000004
Inverse Mills ratio from Creditworth (eq. **??**)	1.4026***	1.4344***
Intercept	9.8845***	9.8930***
N	1664	1664
R^2	0.19	-
F	22.36	20.47

Significance levels: $*$: 10% $**$: 5% $***$: 1%. Column (1) reports probit estimation; column (2) probit estimation taking into account the imputation of data.

Table 8: Marginal effects for the probability of access to credit: bivariate probit with selection (selection equation: probability of being creditworthy)

Dummy=1 if firm located in high-exemption area	-0.0268
Dummy=1 if firm posted collateral	-0.0121***
Dummy=1 if firm posts collateral and is located in high-exemption area	0.0233***
$R_L^B / R_H^{\prime B}$	-1.0617***
Loan original maturity (n. of months)	0.0002***
Amount granted over total applied	0.0460***
Years of firm-bank relationship	0.0005***
Dummy=1 if firm's credit score is in the top 25%	0.0100***
Dummy=1 if firm has delinquency records	-0.0049***
Debts over equity	-0.0001**
Dummy=1 if firm has limited liability	-0.0038**
Total assets - thousands of $	0.000001***
N	1721
Censored observations	130
Uncensored observations	1591
ρ	-0.688
LR test of indep. eqns. ($\rho = 0$)	$\chi^2 = 13.96$

Significance levels: $*$: 10% $**$: 5% $***$: 1%.