Bank Recovery and Resolution Planning, Liquidity Management and Fragility

Luca G. Deidda* Ettore Panetti†

Abstract

We study how regulation shapes the interaction between financial fragility and bank liquidity management, and propose a rationale for the complementarity between bank recovery and resolution planning. To this end, we analyze an economy in which a benevolent resolution authority sets a bank resolution plan according to which it suspends deposit withdrawals and creates a "good bank" at a cost, in the event of a depositors' run. In such a framework, banks maximize expected welfare when they decide ex ante how to manage liquidity during runs. However, this choice is time inconsistent. Therefore, regulators need to force banks to commit to it through recovery planning.

Keywords: banks, liquidity, financial fragility, financial regulation, resolution.
JEL codes: G01, G21, G28

*University of Sassari and CRENoS. Address: Department of Economics and Business, University of Sassari, Via Muroni 25, 07100 Sassari, Italy. Email: deidda@uniss.it

†Department of Economics and Statistics, University of Naples Federico II, Via Cintia 21, 80126 Napoli, Italy, and CSEF, SUERF and UECE-REM. Email: ettore.panetti@unina.it.

1 Introduction

Effective liquidity management is critical for banks, serving as both a preemptive safeguard against financial distress and a means of coping with it. The existing literature has identified fundamental uncertainty as one of the main factors explaining banks' preferences for liquidity (Ashcraft et al., 2011; Acharya and Merrouche, 2013). However, depositors' self-fulfilling expectations are another critical determinant of financial fragility, as the liquidity and maturity transformation at the core of banking leads to a mismatch in banks' balance sheets, creating the potential for depositor runs. Indeed, the evidence of bank runs is not limited to country-specific examples such as Argentina in 2001 and Greece in 2015. The 2007-2009 global financial crisis and the 2010-2012 EU bank and sovereign debt crisis arguably had a significant self-fulfilling component, too (Gorton and Metrick, 2012; Baldwin et al., 2015).

Motivated by such experiences, financial regulators have introduced new ways to deal with financial distress that recognize the critical role of liquidity management. In particular, the Financial Stability Board (2011) has been coordinating international efforts to introduce bank recovery and resolution planning. Such policies provide banks with a framework to address severe financial shocks and unexpected liquidity needs. Moreover, they let governments set the rules for the orderly resolution of financially distressed banks while maintaining the banks' continuity.

In light of these considerations, the aim of this paper is twofold. First, we study the interaction between financial fragility and bank liquidity management. Financial fragility due to self-fulfilling expectations can have a significant impact on liquidity management, as banks plan for potential excessive withdrawals during times of distress by either rolling over liquidity or liquidating productive assets on their balance sheets. In turn, banks' liquidity management shapes investors' perception of their resilience to shocks, which feeds back into financial fragility.

Second, we study how regulation shapes the interaction between financial fragility and bank liquidity management, and provide a rationale for the complementarity of bank recovery and resolution planning. A well-known argument states that resolution plans – in particular, a commitment to a tough suspension of deposit withdrawals – should be sufficient to calm depositors' self-fulfilling expectations of excessive withdrawals (Diamond and Dybvig, 1983). However, in practice, resolution is often delayed because of regulatory costs, political pressure, or imperfect coordination between different regulatory layers.[1] Our main result shows that, in the presence of delayed resolution, banks maximize expected welfare when they choose ex ante how to manage liquidity against self-fulfilling runs. However, such a liquidity management is time inconsistent, as it leaves banks more exposed to runs ex post. Therefore, a regulator willing to maximize expected welfare needs to impose its application through recovery planning.

More in detail, we propose a theory of banking with depositors, banks, and a resolution authority (hereafter, RA). The economy features two types of fundamental uncertainty, namely aggregate productivity shocks that hit banks' investments and idiosyncratic shocks that force depositors to withdraw funds at an interim date, i.e., before the maturity of the investment. Competitive banks collect deposits and offer standard deposit contracts, financed by investing deposits in liquidity and a partially illiquid but productive asset. Due to incomplete contractibility related to idiosyncratic shocks and imperfect information about the aggregate productivity shocks, the economy features multiple equilibria with the possibility of runs by banks' depositors on the interim date. These runs are self-fulfilling because depositors withdraw funds on the interim date only because they expect that all the other depositors also withdraw funds and fear that if they do not do the same, they might end up with zero consumption.[2] We resolve the multiplicity of equilibria following the "global game" approach by Carlsson and van Damme (1993) and Morris and Shin (1998). We assume that the depositors observe a noisy signal about the realization of the economy's aggregate state, based on which they create beliefs about the behavior of all the other depositors and decide whether to withdraw funds on the interim date.

Banks' liquidity management consists of two parts. First, banks choose the initial amount of liquidity in their asset portfolios. Second, they choose the asset pecking order to meet depositors' withdrawals on the interim date. In particular, banks can either first deploy liquidity and then liquidate productive assets, or liquidate productive assets and then deploy liquidity. Both pecking orders entail some costs. On the one hand, deploying liquidity reduces banks' future returns in case of adverse aggregate productivity shocks.

[1]Examples of papers studying the determinants of public intervention on distressed banks includes Repullo (2018), Calzolari et al. (2019), Colliard (2020) and Carletti et al. (2021).

[2]For this argument to hold, we need to assume that there exists no deposit insurance. This assumption can be justified by the growing role of uninsured bank deposits in modern banking systems. In fact, the total amount of uninsured checkable, time, and savings deposits held by U.S. chartered commercial banks in 2020 represented almost 40 percent of total U.S. bank liabilities, after reaching their lowest value of approximately 10 percent in 2009 (source: Financial Accounts of the United States). Furthermore, uninsured deposits represent about half of the total deposits in the largest commercial banks both in the U.S. (Egan et al., 2017) and in the Euro Area (source: ECB Bank Balance Sheet Items and European Banking Authority data).

On the other hand, liquidating productive assets is instead costly in terms of resources lost at liquidation, and forgone future returns in case of positive aggregate productivity shocks.

The RA commits to a resolution plan that maximizes depositors' expected welfare. According to the plan, the RA has the power to suspend deposit withdrawals when a run is underway, and the fraction of depositors withdrawing funds on the interim date reaches a certain threshold. This assumption is consistent with common resolution practices around the world. For example, the European Systemic Risk Board states that "resolution occurs at the point where the authorities determine that a bank is failing or likely to fail, that there is no other supervisory or private sector intervention that can restore the institution to viability (for example by applying measures set out in a so-called recovery plan, which all banks are required to draft) within a short time frame and that normal insolvency proceedings would cause financial instability while having an impact on the public interest.[3]

After suspension, the RA pays a verification cost to establish which depositors need liquidity on the interim date among those who have not already withdrawn funds and reallocates the available resources between them and the remaining depositors. In particular, depositors with early liquidity needs still receive what banks have promised them according to the deposit contract. Accordingly, such a resolution procedure resembles the creation of a "good bank" that shields insured deposits. This is consistent with the recommendations by the Basel Committee on Banking Supervision (2010) and is a crucial part of several resolution plans worldwide. The verification cost instead represents a proxy for the aforementioned mechanisms that might delay bank resolution. In the model, the role of costly verification is also crucial because a credible commitment to a costless suspension of deposit withdrawals could rule out depositors' runs altogether (Diamond and Dybvig, 1983).

We characterize the game's unique symmetric equilibrium between depositors, banks the RA, under two assumptions on the timing of the pecking order decision. First, we analyze the case in which banks choose the asset pecking order ex post on the interim date, after the banks' portfolio and RA's resolution decisions (taken on the initial date). If only the depositors hit by the idiosyncratic shocks withdraw funds, no run occurs. Banks are solvent and use only liquidity to cover depositors' withdrawals without liquidating productive assets. If instead the other depositors not hit by the idiosyncratic shocks also try to withdraw funds, a run occurs. Then, the RA intervenes by suspending withdrawals

[3]Source: https://srb.europa.eu/en/content/what-bank-resolution.

and expropriates banks' liquidity management by creating the good bank. Before the intervention of the RA, we show that banks will always prefer the pecking order by which they first deploy liquidity and then liquidate productive assets. The reason is that under this pecking order, depositors' incentives to run are lower than under the alternative option of first liquidating productive assets and then deploying liquidity. Accordingly, banks recognize this effect and on the initial date choose to hold excess liquidity, i.e., a higher amount of liquidity than the one they would need, to cover depositors' idiosyncratic needs if no runs were to occur.

Second, we compare these results to the equilibrium of an economy in which banks choose the pecking order ex ante on the initial date, at the same time as their portfolio and RA's resolution decisions. In this case, there exists a plausible parameter set under which banks choose a different pecking order: they first liquidate productive assets and then deploy liquidity when facing a depositors' run. This occurs despite the fact that this pecking order brings about a higher likelihood of a run ex post than the alternative option. Banks recognize that their choice of pecking order affects not only the likelihood of a run but also their own asset portfolio and RA's resolution decisions. In particular, under the chosen pecking order, holding more liquidity would increase depositors' incentives to run. Therefore, on the initial date, banks choose to hold no excess liquidity. This in turn counterbalances the costs of a higher likelihood of a run with higher long-term returns from productive investments if a run does not occur.

As a consequence of the previous results, the equilibrium expected welfare can be higher when banks choose the pecking order ex ante rather than ex post. Yet, the ex-ante decision of the pecking order turns out to be time inconsistent. In fact, although it is optimal for banks to announce their intention to first liquidate productive assets and then deploy liquidity when runs occur, once a run is underway it is instead optimal to follow the alternative pecking order because that is the one that minimizes depositors' incentives to run.

The normative conclusion of this argument is that it might be necessary to impose a bank's commitment to the asset pecking order that maximizes expected welfare on the initial date. In other words, in the presence of a regulatory commitment to resolution planning, the time inconsistency of banks' liquidity management during self-fulfilling runs rationalizes the imposition of a recovery plan, that obliges banks to set ex ante their liquidity management at times of distress. The combination of these two regulatory policies has the potential to significantly alter banks' liquidity management in anticipation of self-fulfilling uncertainty and its eventual realization, thus guaranteeing a more valuable allocation of resources and higher welfare.

The present paper contributes to several strands of the literature. Our study is the first to examine banks' asset pecking orders during self-fulfilling depositors' runs in the literature on banks' liquidity and financial fragility. In recent models of bank runs, this issue is completely overlooked. For example, in Goldstein and Pauzner (2005) banks hold no liquidity because it is dominated by the investment in productive assets, which have zero liquidation costs. In contrast, in Rochet and Vives (2004) and Vives (2014) banks do hold liquidity and productive assets, but the asset pecking order during runs is exogenous. Kashyap et al. (2020) develop a model of both sides of banks' balance sheets. However, differently from our study, banks do not hold excess liquidity against self-fulfilling uncertainty and do not choose asset pecking orders. Ahnert and Elamin (2020) also study banks' portfolio choices but assume that banks can liquidate the productive asset at zero cost on the interim date and do not have any liquidity available on the initial date. Moreover, banks have access to liquidity only after depositors' decisions to withdraw funds and eventually run, and after they receive a perfectly informative signal about the realization of the aggregate state.

More generally, our study contributes to the literature on liquidity management of financial institutions prone to self-fulfilling uncertainty. Chen et al. (2010) develop a theory of investors' strategic complementarities to rationalize the empirical evidence on the connection between mutual funds' liquidity and performance. Zeng (2017) studies the optimal rebuilding of cash buffers by open-end mutual funds in a dynamic framework. Liu and Mello (2011) analyze the liquidity management of hedge funds facing coordination risk. Distinctly from all of these papers, our focus is on banks and their maturity transformation and fragility. In this sense, we characterize the optimal asset pecking order, which the cited studies instead all leave to be exogenous.

Finally, our paper contributes to the literature on the economics of bank resolution regimes. Keister and Mitkov (2021) show how the RA's lack of commitment in resolving banks leads to banks' delay in bail-ins. Colliard and Gromb (2018) instead explore the effect of the government's involvement in the private restructuring of distressed banks. Schilling (2018) examines the optimal delay of bank resolution and its effects on financial fragility. These studies do not analyze the consequences of recovery and resolution planning for banks' choices of asset pecking orders and liquidity management as we do.

The rest of the paper is organized as follows. Section 2 states the basic features of the environment. Section 3 studies banks' liquidity management under an ex-post optimal pecking order. Section 4 describes the equilibrium under and ex-ante optimal pecking order and analyzes its time inconsistency. Finally, Section 5 concludes the paper.

2 Environment

The economy lasts for three dates, labeled $t = 0, 1, 2$, and is populated by a unitary continuum of ex-ante identical depositors, a large number of banks, and a resolution authority (RA). Each depositor is endowed with 1 unit of a consumption good on date 0, and 0 afterward. On date 1, all depositors are hit by a privately observed idiosyncratic shock θ, of value 0 with probability λ and value 1 with probability $1-\lambda$. The law of large numbers holds. Hence, the probability distribution of the shocks is equivalent to the cross-sectional distribution of their realization: on date 1, there is a fraction λ of depositors in the whole economy who experience the realized shock $\theta = 0$, and a fraction $1 - \lambda$ of depositors who experience the realized shock $\theta = 1$. Depositors are risk-neutral, and each depositor's individual utility function is $U(c_1, c_2, \theta) = (1 - \theta)c_1 + \theta c_2$. Accordingly, the idiosyncratic shock θ affects the point in time when depositors want to consume. Depositors experiencing the shock $\theta = 0$ are only willing to consume on date 1, and those experiencing the shock $\theta = 1$ are only willing to consume on date 2. Consistently with the literature, we refer to these two categories as early and late consumers, respectively.

There are two technologies available in the economy. The first is a storage technology, which we call "liquidity". It yields 1 unit of consumption on date $t + 1$ for each unit invested on date t. The second is a productive asset that, for each unit invested on date 0, yields a stochastic return Z on date 2. The stochastic return takes values $R > 1$ with probability p, and 0 with probability $1 - p$, and its realization is publicly revealed at the beginning of date 2. The probability of success p represents the aggregate state of the economy and is uniformly distributed over the interval $[0, 1]$, with $\mathbb{E}[p]R > 1$. The productive asset can be liquidated on date 1 via a liquidation technology that allows to recover $r < 1$ units of consumption for each unit of liquidation. In other words, the economy features a liquid asset with a low but safe yield, and an illiquid asset that yields a low return in the short run, and a possibly high return in the long run, subject to the realization of an aggregate productivity shock.

Banks operate in a competitive market with free entry. They offer to the depositors a standard deposit contract, that allows them to retrieve their deposits on date 1 upon demand. In other words, the contract states an amount of early consumption $c = 1$ that depositors can withdraw on date 1, and an amount $c_2(Z)$ that they can withdraw on date 2. The banks' offer of a noncontingent amount of early consumption depends on the true realization of the aggregate state being revealed only in the beginning of date

2. Moreover, the assumption that early consumption is $c = 1$ allows us to maintain in a parsimonious way a rationale for liquidity management in the presence of risk neutrality.[4]

At date 0, the banks collect the deposits – the only liability on their balance sheets – and invest them in a portfolio of L units of liquidity and $1 - L$ units of the productive asset. Then, the banks on date 1 pay early consumption to all the depositors who demand funds early until the banks' resources are depleted. If resources are not exhausted at date 2, the depositors who have not withdrawn on date 1 are residual claimants on equal shares of the remaining resources on date 2, so the amount of late consumption $c_2(Z)$ is the one that clears banks' budget.

Banks also choose an asset pecking order to cover withdrawals on date 1. Under the pecking order {Liquidation, Liquidity}, banks first liquidate the productive asset in portfolio and then deploy the available liquidity. Under the pecking order {Liquidity, Liquidation}, they instead first deploy liquidity and then liquidate the productive asset. We will compare outcomes under two different assumptions about timing: one in which the pecking order is chosen ex post on date 1, as a run unfolds, and one in which it is chosen ex ante on date 0. In what follows, we denote the pecking orders {Liquidation, Liquidity} and {Liquidity, Liquidation} by subscripts (1) and (2), respectively.

We assume that the depositors cannot observe the true value of the realization of the aggregate state of the economy p, but receive a signal $\sigma = p + e$ about it on date 1. The term e is an idiosyncratic noise, uniformly distributed over the interval $[-\epsilon, +\epsilon]$, where ϵ is positive but arbitrarily close to zero. Given the received signal, late consumers decide whether to wait and withdraw funds from the bank on date 2 or "run on the bank" and withdraw funds on date 1. In particular, we assume that late consumers follow a threshold strategy: they run if the signal they receive is lower than a threshold signal σ^*. This strategy is based on the expected advantage of waiting over running, which explicitly depends on depositors' beliefs and bank asset portfolio.

Finally, the RA maximizes depositors' expected welfare by choosing a resolution plan on date 0. A resolution plan is characterized by two features. First, the RA chooses a suspension point, i.e., the fraction ϕ of depositors that are allowed to withdraw on date 1 before suspending withdrawals. Second, after suspension, the RA creates a "good bank". The good bank observes the types of depositors (early or late) that have not participated in the run after paying a verification cost $\psi(\phi)$, decreasing in the fraction

[4]In fact, with a linear utility banks in equilibrium would choose the lowest possible value of early consumption. Carletti et al. (2021), who also study a banking model with a linear utility, encounter the same issue and adopt the same assumption.

ϕ of depositors who have withdrawn before suspension. Then, it pays $c = 1$ to early consumers and equally shares the resources left among the remaining late consumers who have not withdrawn early.

We solve the model by backward induction and characterize a pure-strategy symmetric perfect Bayesian equilibrium. Hence, we focus our attention on the behavior of a representative bank, hereafter referred to as “the bank”. The definition of equilibrium is as follows:

Definition 1. *Given the distributions of idiosyncratic shocks θ, the aggregate productivity shock Z and the individual signals σ, an equilibrium comprises an asset portfolio $\{L, 1-L\}$ and a pecking order chosen by the bank, a set of depositors' withdrawal decisions and a suspension point ϕ decided by the RA such that, conditional on beliefs*

- *The depositors' withdrawal decisions maximize their expected welfare;*
- *The bank's pecking order and asset portfolio maximize the depositors' expected welfare, subject to the bank's budget constraint;*
- *The RA's suspension point maximizes the depositors' expected welfare, subject to the bank's budget constraints, and*
- *The bank's, depositors' and RA's beliefs are updated according to the strategies used and the Bayes' rule.*

In a benchmark economy with perfect information, the representative bank can observe and verify the realizations of the idiosyncratic shocks hitting the depositors but not the realization of the aggregate state. In Appendix B, we show that in the equilibrium of this economy, there is no liquidation of the productive asset and no excess liquidity, i.e., $D = 0$ and $L = \lambda$. This is because liquidating productive assets to create liquidity on date 1 is too costly as the recovery rate r is smaller than 1. Consequently, with perfect information, the bank only uses liquidity to cover withdrawals on date 1. Moreover, the advantage of holding excess liquidity, in terms of consumption in the bad state of the world when the productive asset yields zero, is dominated by its cost in terms of forgone return in the good state of the world. Hence, the bank in equilibrium holds just enough liquidity to cover the total early consumption λ.

3 Equilibrium with ex-post optimal pecking order

We start by analyzing the equilibrium of the game assuming that the bank chooses the asset pecking order to meet its liquidity needs ex post on date 1, as a run unfolds and after its portfolio and RA's resolution decisions. The timing of the actions is as follows.

- On date 0, the bank chooses the asset portfolio $\{L, 1-L\}$. Based on this information, the RA chooses the suspension point ϕ, which is part of the resolution plan;
- On date 1, all depositors become aware of their private types and signals and decide whether to withdraw funds. Then, those who decide to withdraw take positions in line and start withdrawing funds until resources are exhausted. The bank chooses the asset pecking order to meet depositors' withdrawals, while the RA eventually suspends withdrawals once the suspension point ϕ is reached, pays the verification cost, and creates a good bank.
- On date 2, those late consumers who have not withdrawn funds on date 1 withdraw equal shares of the available resources left.

To characterize the equilibrium, we first study the consumers' withdrawal decisions on date 1, for given pecking order and resolution plan. Then, we study the bank's choice of the pecking order on date 1, and finally, the bank's choice of the asset portfolio and the RA's choice of the suspension point on date 0.

The depositors observe their private types and signals, and decide whether to withdraw funds on date 1. Early consumers certainly want to do that, while late consumers choose whether to join them (i.e., "run on the bank") or wait until date 2. Early withdrawers arrive at the bank in random order and are served sequentially. This means that the bank and the RA do not know that a run is underway until the fraction of depositors withdrawing funds exceeds λ. Hence, the bank serves the first λ early withdrawers with liquidity as in the equilibrium with perfect information, and chooses the asset pecking order to cover further withdrawals.

3.1 Pecking orders and depositors' withdrawal decisions

The presence of noisy signals allows us to apply global-game techniques. Late consumers act based on their private signals σ on date 1, and take as given the RA's resolution plan and bank asset portfolio fixed on date 0. Based on this information, they create posterior beliefs about the probability of the realization of the aggregate productivity shock Z and about the number n of depositors withdrawing funds at time 1, and decide whether to join them.

We assume the existence of two extreme regions such that, for signal realizations within them, the withdrawal decisions of late consumers are independent of such consumers' posterior beliefs. Following Goldstein and Pauzner (2005), we assume an "upper dominance region" above a threshold $\overline{\sigma}$ where the productive asset is safe, i.e., $p = 1$, and yields the same return R on dates 1 and 2. In this way, late consumers who wait are sure to receive $(R(1-L)+RL-n)/(1-n) > 1$ on date 2. Hence, they will never run for any fraction n of depositors withdrawing early. In the "lower dominance region" instead, the signal is so low that late consumers always withdraw funds on date 1 irrespective of the behavior of the other depositors. This occurs below the threshold signal $\underline{\sigma}$ that makes late consumers indifferent between withdrawing or not:

$$1 = \underline{\sigma}\frac{R(1-L)+L-\lambda}{1-\lambda} + (1-\underline{\sigma})\frac{L-\lambda}{1-\lambda}. \tag{1}$$

Hence:

$$\underline{\sigma}(L) = \frac{1-L}{R(1-L)} = \frac{1}{R}. \tag{2}$$

The existence of the lower and upper dominance regions ensures the existence of an equilibrium in the intermediate region $[\underline{\sigma}, \overline{\sigma}]$, where late consumers decide whether to run or not based on their beliefs about the true realization of the aggregate state and other depositors' actions.

To characterize depositors' withdrawal behavior in the intermediate region, we first study the utility advantage of waiting over running under both pecking orders, for a given fraction n of depositors who withdraw funds on date 1 and for a given suspension point ϕ. Table 1 reports payoffs under both pecking orders. In Table 1a (pecking order {Liquidation, Liquidity}) if the fraction of depositors who withdraw funds on date 1 is in the interval (λ, ϕ) (i.e., before suspension) the bank fulfills its contractual obligation on date 1 by liquidating productive assets first. It needs to provide an amount equal to 1 of early consumption to $n-\lambda$ depositors using resources rD from liquidation. Hence, the amount of productive assets to be liquidated is $D = (n-\lambda)/r$. Then, if n depositors withdraw funds on date 1, consumption of a late consumer who waits until date 2 is

$$c_L(Z, n) = \frac{Z\left(1 - L - \frac{n-\lambda}{r}\right) + L - \lambda}{1-n}, \tag{3}$$

Table 1: Depositors' ex-post payoffs.

(a) Pecking order {Liquidation, Liquidity}

Date	$n \in (\lambda, \phi)$	$n \in [\phi, 1]$	
$t = 1$	1	1	
$t = 2$	$\frac{Z\left(1-L-\frac{n-\lambda}{r}\right)+L-\lambda}{1-n}$	$\frac{Z\left(1-L-\frac{\phi-\lambda}{r}\right)+L-\lambda-(1-\phi)\lambda}{(1-\lambda)(1-\phi)}$	$\forall Z \in \{0, R\}$

(b) Pecking order {Liquidity, Liquidation}

Date	$n \in (\lambda, n_2^*)$	$n \in [n_2^*, \phi)$	$n \in [\phi, 1]$	
$t = 1$	1	1	1	
$t = 2$	$\frac{Z(1-L)+L-n}{1-n}$	$\frac{Z\left(1-L-\frac{n-L}{r}\right)}{1-n}$	$\frac{Z\left(1-L-\frac{\phi-L}{r}-\frac{(1-\phi)\lambda}{r}\right)}{(1-\lambda)(1-\phi)}$	$\forall Z \in \{0, R\}$

depending on the realization of the aggregate productivity shock $Z \in \{0, R\}$, as the remaining liquidity $L - \lambda$ is rolled over from date 1 to date 2.

If the fraction of depositors who withdraw funds on date 1 is in the interval $[\phi, 1)$ (i.e., after suspension) the good bank created by the RA holds an amount $1 - L - (\phi - \lambda)/r$ of productive assets yielding a return $Z \in \{0, R\}$, and excess liquidity $L - \lambda$. The good bank uses excess liquidity to serve the remaining $(1 - \phi)\lambda$ early consumers who were not served before suspension, and distributes to them an amount 1 of resources according to the deposit contract. Note that the good bank would never liquidate productive assets at this stage because, after paying the verification cost, is fully informed about depositors' types. Hence, it follows the same strategy as in perfect information. The good bank then uses the remaining liquidity and the proceeds from the productive assets to serve the remaining $(1 - \lambda)(1 - \phi)$ late consumers on date 2.

In Table 1b (pecking order {Liquidity, Liquidation}) instead, $n_2^* = L$ is the maximum fraction of depositors that a bank can serve on date 1 using liquidity, given early consumption equal to 1. If the fraction of depositors who withdraw funds on date 1 is in the interval (λ, n_2^*), the bank fulfills its contractual obligation by using liquidity, while retaining the productive asset. Hence, in this case, the consumption of a late consumer who waits until date 2 is $c_L(R, n) = (Z(1 - L) + L - n)/(1 - n)$ depending on the realization of the aggregate productivity shock $Z \in \{0.R\}$. If the fraction of depositors who withdraw funds on date 1 is instead in the interval $[n_2^*, \phi)$, the bank is forced to fulfill its contractual obligation by also liquidating the productive assets in its portfolio. Hence, the total resources available to provide early consumption equal to 1 are $L + rD$, and the amount that the bank liquidates is equal to $D = \frac{n-L}{r}$. Moreover, as the liquidity has been exhausted, the consumption of a late consumer who waits until date 2 is

$$c_L^D(Z, n) = \frac{Z\left(1 - L - \frac{n-L}{r}\right)}{1-n}, \tag{4}$$

for any $Z \in \{0, R\}$. Finally, if the fraction of depositors who withdraw funds on date 1 is in the interval $[\phi, 1]$ (i.e., after suspension) the good bank holds an amount $1-L-(\phi-L)/r$ of productive assets, and further liquidates an amount $(1-\phi)\lambda/r$ to serve early consumers. As before, the remaining resources are equally split among the remaining $(1-\lambda)(1-\phi)$ late consumers on date 2.

Given the described payoff structure, under the pecking order {Liquidation, Liquidity} the utility advantage of waiting over running before suspension, i.e., when $n \in (\lambda, \phi)$, is

$$v_1(\sigma, n) = \sigma \frac{R\left(1 - L - \frac{n-\lambda}{r}\right) + L - \lambda}{1-n} + (1-\sigma)\frac{L-\lambda}{1-n} - 1, \tag{5}$$

for $\lambda < n < \phi$, while under the pecking order {Liquidity, Liquidation} it is

$$v_2(\sigma, n) = \begin{cases} \sigma \frac{R(1-L)+L-n}{1-n} + (1-\sigma)\frac{L-n}{1-n} - 1 & \text{if } \lambda < n < n_2^*, \\ \sigma \frac{R(1-L-D)}{1-n} - 1 = \sigma \frac{R\left(1-L-\frac{n-L}{r}\right)}{1-n} - 1 & \text{if } n_2^* \leq n < \phi. \end{cases} \tag{6}$$

Figure 1 plots the two functions and highlights the role of suspension in this framework. Under both pecking orders, we guess that the verification costs are so high that both $v_1(\sigma, \phi)$ and $v_2(\sigma, \phi)$ are negative, meaning that suspension arrives too late to stop a run. In fact, if $v_1(\sigma, \phi)$ and $v_2(\sigma, \phi)$ were positive, the value of waiting would exceed the value of running for any $n < \phi$. Thus, no late consumer would run. At the same time, we guess that the verification costs are sufficiently low such that, under the pecking order {Liquidation, Liquidity}, the suspension arrives before the bank has liquidated all productive assets, i.e., $\phi \leq \lambda + r(1-L)$. Section 4.2 provides a numerical characterization of the equilibrium, that verifies these guesses for a reasonable parameter space.[5]

[5] As a further check on the plausibility of the guesses, we also numerically characterize a different equilibrium, in which we guess that under the pecking order {Liquidation, Liquidity} the suspension arrives after the bank has liquidated all productive assets. In such a framework, we are able to show that in the same parameter space as before the guess is not verified.

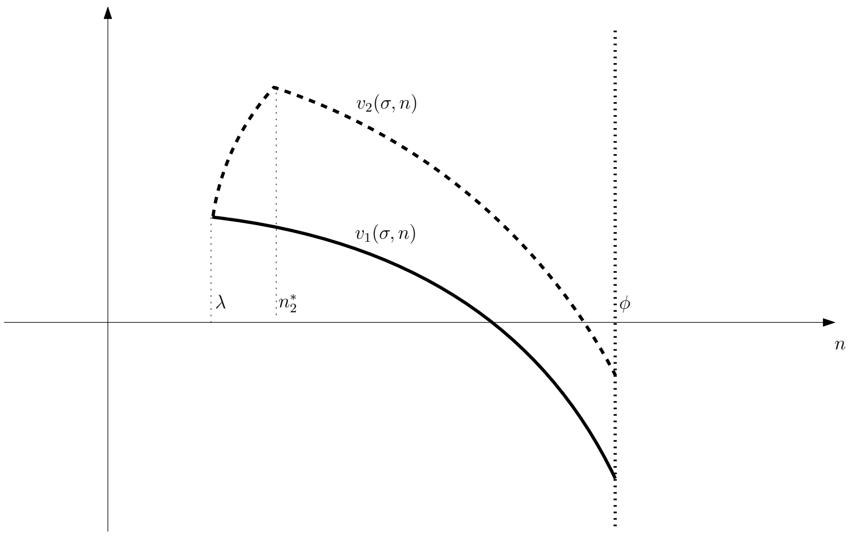

Figure 1: The advantage of waiting over running, as a function of the fraction of depositors running n, under the pecking orders {Liquidation, Liquidity} (solid line) and {Liquidity, Liquidation} (dashed line). The dotted line indicates the suspension point ϕ.

The strategic complementarities affecting a late consumer's decision to run depend on how the advantage of waiting over running varies with the fraction of depositors n withdrawing funds on date 1.

Lemma 1. *The function $v_1(\sigma, n)$ is decreasing in $n \in (\lambda, \phi)$. The function $v_2(\sigma, n)$ is increasing in $n \in (\lambda, n_2^*)$ and decreasing in $[n_2^*, \phi)$.*

Proof. In Appendix A. ■

Figure 1 shows that under the pecking order {Liquidation, Liquidity} the economy exhibits strategic complementarities, i.e., the advantage of waiting over running is decreasing in the fraction of depositors running before suspension.

In contrast, under the pecking order {Liquidity, Liquidation} the advantage of waiting over running is increasing in the fraction of depositors running as long as the bank holds sufficient liquidity to serve them, i.e., in the interval (λ, n_2^*). This occurs because the positive effect on the incentives to wait given by the lower number of depositors who wait dominates the negative effect given by the lower amount of liquidity rolled over from date 1. Yet, strategic complementarities emerge for n in the interval $[n_2^*, \phi)$. This is because as n increases the bank is forced to liquidate a higher and higher amount of productive assets, therefore lowering the date-2 payoff and increasing late consumers' incentives to run.

In any case, both functions $v_1(\sigma, n)$ and $v_2(\sigma, n)$ cross zero only once, and are increasing in σ. Altogether, these properties guarantee the existence and uniqueness of the equilibrium under both pecking orders (Goldstein and Pauzner, 2005).

3.2 Ex-post optimal pecking order

By backward induction, now we characterize the bank's decision of the pecking order as the run unfolds, i.e., at any possible fraction n of depositors running before resolution occurs. Two cases emerge. First, suppose that no depositor runs, i.e., $n = \lambda$. In this case, the outcome is equivalent to that of perfect information, where the bank never liquidates the productive asset, as shown in Appendix B. Second, if a run occurs, the RA suspends withdrawals after ϕ depositors have been served, and creates a good bank. In this case, the bank still has to choose the pecking order to serve the depositors before suspension, i.e., when $n \in (\lambda, \phi)$.

Proposition 1. *If $n \in (\lambda, \phi)$, the pecking order $\{$Liquidity, Liquidation$\}$ is always preferred to $\{$Liquidation, Liquidity$\}$.*

Proof. In Appendix A. ■

The intuition for this result is as follows. A bank willing to maximize expected welfare by choosing the pecking order ex post (i.e., after its portfolio and RA's suspension decisions) should pick the one that minimizes depositors' incentives to run. As a run unfolds, there are two ways in which the bank can pay one unit of early consumption: deploy one unit of liquidity, or liquidate $1/r$ units of the productive asset. The second option entails a loss of expected return on date 2 and a loss from costly liquidation, hence it is more expensive than the first option. For any $n \in (\lambda, \phi)$, under {Liquidity, Liquidation} the bank has to liquidate a lower amount of productive assets to serve early withdrawers than under {Liquidation, Liquidity}. Therefore, late consumers' payoffs are higher under {Liquidity, Liquidation} than under {Liquidation, Liquidity}. Put differently, the pecking order {Liquidity, Liquidation} is the one that induces lower depositors' incentives to run.

Proposition 1 implies that the threshold signal σ^* that determines depositors' withdrawal strategies is the one that makes depositors indifferent between running or not given their beliefs under the pecking order {Liquidity, Liquidation}. Put differently, it is the value of σ that solves $\mathbb{E}[v_2(\sigma,n)|\sigma_2^*]=0$, i.e.,

$$\sigma_2^*(L,\phi)=\frac{\phi-\lambda-\int_\lambda^{n_2^*}\frac{L-n}{1-n}dn}{\int_\lambda^\phi \frac{R(1-L)}{1-n}dn-\int_{n_2^*}^\phi \frac{R}{r}\frac{n-L}{1-n}dn}. \tag{7}$$

This threshold defines the probability of a run in this framework, as by the uniform distribution of the signals prob$\{\sigma\le\sigma_2^*\}=\sigma_2^*$. A comparative statics exercise clarifies the different channels through which bank liquidity L and the suspension point ϕ affect this threshold:

$$\frac{\partial\sigma_2^*}{\partial L}=\frac{1}{DEN_{\sigma_2^*}}\left[-\int_\lambda^{n_2^*}\frac{1}{1-n}dn-\sigma_2^*R\left(-\int_\lambda^\phi\frac{1}{1-n}dn+\frac{1}{r}\int_{n_2^*}^\phi\frac{1}{1-n}dn\right)\right], \tag{8}$$

$$\frac{\partial\sigma_2^*}{\partial\phi}=\frac{1}{DEN_{\sigma_2^*}}\left[1-\sigma_2^*\frac{R\left(1-L-\frac{\phi-L}{r}\right)}{1-\phi}\right], \tag{9}$$

where $DEN_{\sigma_2^*}$ is the denominator of σ_2^*. As far as the effect of liquidity is concerned, two forces are at play. On the one hand, higher liquidity implies higher excess liquidity (the first term of (8)) and reduces the need to liquidate productive assets (the third term of (8)), and these increase depositors' incentives to wait. On the other hand, higher liquidity reduces the investment in the productive asset (the second term of (8)), and therefore the depositors' incentives to wait. Therefore, the total effect is ambiguous.

As far as suspension is concerned, observe that the later it arrives, the fewer resources are available for late consumption, and therefore the lower the depositors' incentives to wait. This implies that the RA wants to suspend withdrawals as soon as possible: in principle, it could announce a suspension before the advantage of waiting over running $v_2(\sigma,n)$ becomes negative, and rule out runs altogether. However, by suspending withdrawals the RA incurs the verification costs to create a good bank, and this might postpone the intervention.

3.3 Bank's asset portfolio and RA's suspension decisions

Having completed the characterization of the depositors' withdrawals strategies and asset pecking order on date 1, by backward induction we now solve for the asset portfolio and the suspension point chosen by the bank and the RA on date 0, respectively. Both the bank and the RA maximize depositors' expected welfare:

$$W_2(L,\phi) = \int_0^{\sigma_2^*(L,\phi)} \left[\phi + (1-\phi)\left[\lambda + (1-\lambda)p\frac{R\left(1-L-\frac{\phi-L}{r}-\frac{(1-\phi)\lambda}{r}\right)}{(1-\lambda)(1-\phi)}\right] - \psi(\phi)\right] dp +$$
$$+ \int_{\sigma_2^*(L,\phi)}^1 \left[\lambda + (1-\lambda)\left[p\frac{R(1-L)+L-\lambda}{1-\lambda} + (1-p)\frac{L-\lambda}{1-\lambda}\right]\right] dp \tag{10}$$

subject to $\lambda \leq L \leq 1$. If $p \leq \sigma_2^*$, a run occurs and suspension takes place:[6] ϕ depositors receive $c = 1$ before suspension; among the $(1-\phi)$ depositors served after suspension by the good bank, λ are early consumers and also receive $c = 1$, and the remaining $1-\lambda$ are late consumers and receive an equal share of the available resources on date 2. If instead the signal is above the threshold σ_2^*, a run does not occur. The fraction λ of depositors who are early consumers receive $c = 1$, while the fraction $1-\lambda$ of depositors are late consumers who consume either $(R(1-L)+L-\lambda)/(1-\lambda)$ or $(L-\lambda)/(1-\lambda)$, depending on whether the productive asset yields a positive or a zero return, respectively.

To characterize liquidity and the suspension point, define the difference between the utility in the no-run case and the utility in the case of a run as

$$\Delta U_2 = L + \sigma_2^* R(1-L) - \phi - (1-\phi)\lambda - \sigma_2^* R\left(1-L-\frac{\phi-L}{r}-\frac{(1-\phi)\lambda}{r}\right) + \psi(\phi). \tag{11}$$

Then, the first-order condition of the above optimization problem with respect to ϕ yields

$$-\sigma_2^*(L,\phi)\psi'(\phi) = (1-\lambda)\left[\frac{\sigma_2^{*2}(L,\phi)R}{2r} - \sigma_2^*(L,\phi)\right] + \frac{\partial\sigma_2^*}{\partial\phi}\Delta U_2. \tag{12}$$

This expression states that the RA chooses the equilibrium suspension point ϕ so as to equalize its expected marginal costs, in terms of the verification costs that it needs to pay to set a good bank at resolution (the left-hand side of (12)), and its expected marginal benefits, in terms of the lower amount of productive assets to liquidate and lower probability of a run.

The first-order condition with respect to L yields

$$\frac{\sigma_2^{*2}}{2}R\left(\frac{1}{r}-1\right) + \int_{\sigma_2^*}^1 \left[-pR+1\right]dp - \frac{\partial\sigma_2^*}{\partial L}\Delta U_2 + \xi - \chi = 0, \tag{13}$$

[6] This holds since we focus on the case of vanishing signal noises.

where ξ and χ are the respective Lagrange multipliers on the constraints $\lambda \leq L \leq 1$. Compared to the equilibrium with perfect information, in this case, the bank takes into account two more effects of holding liquidity. First, higher liquidity allows the bank to postpone asset liquidation, thus leaving more resources to the good bank in case of resolution (the first term of (13)). Second, it affects depositors' incentives to run, as represented by the marginal effect of L on $\sigma_2^*(L, \phi)$ (the third term of (13)). While the former effect is unquestionably positive, the latter has an ambiguous sign, as shown in (8). In other words, it is not clear whether the bank always holds excess liquidity in equilibrium. The following proposition provides a sufficient condition for this to happen.

Proposition 2. *In the equilibrium with ex-post optimal pecking order, if $r < 1/R^2$ the bank holds excess liquidity, i.e., $L > \lambda$.*

Proof. In Appendix A. ■

The intuition for this result is straightforward. If $L = \lambda$, under the pecking order {Liquidity, Liquidation} financial fragility is decreasing in liquidity as the first channel analyzed in (8) dominates the second. Then, if the recovery rate is sufficiently small, the marginal benefit of holding liquidity, in terms of postponing asset liquidation at resolution, dominates its marginal cost, in terms of lower investment in the productive asset. This means that there are unexploited marginal benefits from liquidity, and $L = \lambda$ cannot be an equilibrium.

4 Equilibrium with ex-ante optimal pecking order

In this section, we proceed to the analysis of the game assuming that, differently from the previous section, the bank chooses the pecking order ex ante on date 0, at the same time as its own portfolio and RA's suspension decisions. Such timing corresponds to the idea that banks choose a recovery plan on date 0, which specifies the pecking order to manage liquidity if a run unfolds. The timing of actions is otherwise the same as that of the game analyzed in the previous section. As before, we solve the model by backward induction. Hence, we start from the characterization of depositors' withdrawal strategies σ_j^* for a given suspension point, bank asset portfolio and pecking order $j \in \{1, 2\}$. For this reason, we label suspension point and liquidity with a subscript that keeps track of the pecking order under which they are chosen. Then, we solve for the equilibrium suspension point for a given bank asset portfolio and pecking order, and finally characterize the bank's decisions on date 0. Put differently, we solve for the depositors' withdrawal strategies, suspension point and asset portfolio under each pecking order separately, and then let the bank pick the pecking order that maximize expected welfare on date 0.

The analysis of the depositors' withdrawal strategies, suspension point and bank's asset portfolio under the pecking order {Liquidity, Liquidation} is the same as in the previous section. Therefore, here we only need to solve for the case of {Liquidation, Liquidity}. As a first step, we characterize the threshold signal σ_1^*. Formally, the threshold is the value of σ that solves $\mathbb{E}[v_1(\sigma, n)|\sigma_1^*] = 0$ where $v_1(\sigma, n)$ is defined in (42), i.e.,

$$\sigma_1^*(L_1, \phi_1) = \frac{\phi_1 - \lambda - \int_\lambda^{\phi_1} \frac{L_1-\lambda}{1-n} dn}{\int_\lambda^{\phi_1} \frac{R\left(1-L_1-\frac{n-\lambda}{r}\right)}{1-n} dn}. \tag{14}$$

Similarly to the analysis of {Liquidity, Liquidation}, this threshold characterizes the probability of a run under the pecking order {Liquidation, Liquidity}. The following lemma shows how this is affected by the RA's choice of suspension point and the bank's liquidity.

Lemma 2. *Under {Liquidation, Liquidity}, the probability of a run is increasing in the suspension point ϕ_1 and in liquidity L_1.*

Proof. In Appendix A. ■

Intuitively, as the suspension arrives too late to stop a run, the later it arrives, the smaller the resources available for late consumption after suspension, and therefore the higher the depositors' incentive to run. The intuition is similar as far as liquidity is concerned: the higher liquidity is, the smaller the investment in productive assets, and therefore the higher the depositors' incentives to run.

4.1 Asset portfolio and suspension under {Liquidation, Liquidity}

By backward induction, we now study the equilibrium bank asset portfolio and the optimal suspension point under the pecking order {Liquidation, Liquidity}. As before, the bank and the RA maximize depositors' expected welfare:

$$W_1(L_1, \phi_1) = \int_0^{\sigma_1^*(L_1,\phi_1)} \left[\phi_1 + (1-\phi_1)\left[\lambda + (1-\lambda)\left[p\frac{R\left(1-L_1-\frac{\phi_1-\lambda}{r}\right)}{(1-\phi_1)(1-\lambda)} + \frac{L_1-\lambda-(1-\phi_1)\lambda}{(1-\phi_1)(1-\lambda)}\right]\right]\right.$$

$$-\psi(\phi_1)\Big] dp + \int_{\sigma_1^*(L_1,\phi_1)}^1 \left[\lambda + (1-\lambda)\left[p\frac{R(1-L_1)+L_1-\lambda}{1-\lambda} + (1-p)\frac{L_1-\lambda}{1-\lambda}\right]\right] dp, \tag{15}$$

subject to $\lambda \le L \le 1$. If $p \le \sigma_1^*$, a run occurs and a suspension takes place: ϕ_1 depositors receive $c = 1$ before suspension; among the $(1-\phi_1)$ depositors served after suspension, λ are early consumers and also receive $c = 1$, and the remaining $1-\lambda$ are late consumers and receive an equal share of the remaining resources. Notice that, as explained above, the good bank after suspension does not liquidate assets to pay early consumers, as it is fully informed about depositors' types. Therefore, late consumers share the proceeds from the unliquidated productive assets $1 - L_1 - (\phi_1 - \lambda)/r$ plus whatever excess liquidity is left after serving early consumers, i.e., $L_1 - \lambda - (1-\phi_1)\lambda$.

To characterize liquidity and suspension point, as before we first define the difference between the utility in the case of no-run and the utility in the case of run as:

$$\Delta U_1 = \sigma_1^* R(1-L_1) - \phi_1 - \sigma_1^* R\left(1 - L_1 - \frac{\phi_1 - \lambda}{r}\right) + \lambda + \psi(\phi_1). \tag{16}$$

Then, the first-order conditions with respect to ϕ_1 and L_1 yield the equilibrium conditions for the optimal suspension point and bank's liquidity as:

$$-\sigma_1^*(L_1,\phi_1)\psi'(\phi_1) = \left[\frac{\sigma_1^{*2}(L_1,\phi_1)R}{2r} - \sigma_1^*(L_1,\phi_1)\right] + \frac{\partial\sigma_1^*}{\partial\phi_1}\Delta U_1, \tag{17}$$

$$\int_0^{\sigma_1^*}\big[-pR+1\big]dp + \int_{\sigma_1^*}^1\big[-pR+1\big]dp - \frac{\partial\sigma_1^*}{\partial L_1}\Delta U_1 + \xi - \chi = 0, \tag{18}$$

where ξ and χ are Lagrange multipliers. Under {Liquidation, Liquidity}, the RA chooses the suspension point so as to equalize its expected marginal costs and marginal benefits. The bank chooses the amount of liquidity to hold taking into account that it would like to hold no excess liquidity if no run occurs. Moreover, higher liquidity increases depositors' incentives to run (the third term of (18) is positive as proved in Lemma 2) and the resources available to the good bank after suspension. Notice that the good bank has no incentives to hold liquidity and lower the holding of productive assets until maturity, i.e., the third term of (18) is negative. Hence, the following holds:

Proposition 3. *Under {Liquidation, Liquidity}, the bank holds no excess liquidity, i.e.* $L_1 = \lambda$.

Proof. In the text above. ■

Put differently, the bank might want to tilt its liquidity holding away from the minimum amount that it would need to serve early consumers, in order to try to affect the run probability. However, under {Liquidation, Liquidity} the run probability is increasing in liquidity. Hence, the bank finds it optimal to hold no excess liquidity. This, together with the equilibrium condition for the optimal suspension point in (17), allows us to further derive the following comparative statics result:

Corollary 1. *Under {Liquidation, Liquidity}, the equilibrium suspension point* ϕ_1 *is independent of the return on the productive asset* R*.*

Proof. In Appendix A. ■

The intuition for this is straightforward. As the bank does not hold excess liquidity, the only way in which the return on the productive asset can affect the equilibrium suspension point is by directly influencing RA's incentives. This happens in two ways. On the one hand, R affects the expected marginal cost of suspension (the left-hand side of (17)) through the threshold signal σ_1^*. On the other hand, R also affects the marginal benefit of the suspension (the right-hand side of (17)) through the marginal effect of anticipating suspension on the run probability, and by lowering the amount of productive assets that need to be liquidated in case of a run. The lemma shows that these two channels perfectly offset each other.

4.2 Ex-ante optimal pecking order

Having characterized depositors' withdrawal strategies, RA's choice of suspension point, and the bank's asset portfolio under the two pecking orders separately, we are left with the choice of the ex-ante optimal pecking order. On date 0, the bank chooses the pecking order that maximizes depositors' expected welfare. As a consequence, the equilibrium welfare with ex-ante optimal pecking order is

$$W = \max \{W_1(L_1, \phi_1), W_2(L_2, \phi_2)\}, \tag{19}$$

where notice that $W_2(L_2, \phi_2)$ is the expected welfare under pecking order {Liquidity, Liquidation} that comes from the solution of the problem in Section 3.

As a closed-form solution to this maximization problem is not possible, we propose a numerical solution. Assume quadratic verification costs for the RA of the form $\psi(\phi_j) = (1-\phi_j)^2/2$ for both pecking orders $j \in \{1, 2\}$. We fix the probability of being an early consumer λ to 0.02, as in Mattana and Panetti (2021). We solve the model for a return

Table 2: Equilibrium with ex-ante optimal pecking order.

Pecking order	σ^*	ϕ	L	W_S	W_L	W_A	W
{Liquidation, Liquidity}	0.6246	0.7591	0.0200	0.4675	0.0075	0.6096	1.0846
{Liquidity, Liquidation}	0.4967	0.7234	0.5972	0.3837	0.3006	0.3095	0.9938

on the productive asset $R = 2.04$ so that the expected net return is 2 percent, which is roughly the average return on bank credit to nonfinancial corporations during the period 2003-2019 in the Euro Area.[7] Finally, we select a recovery rate $r = 0.80$, as found by Acharya et al. (2007). Table 2 reports the probability of a run, suspension point, and liquidity under the two pecking orders. Moreover, the table shows depositors' expected welfare W under the two pecking orders, and its decomposition in three parts: the welfare W_S from the good bank after suspension when a run occurs; the welfare W_L from liquidity when a run does not occur; the welfare W_A from the productive assets when a run does not occur.

The numerical results confirm Proposition 3: under {Liquidation, Liquidity} the bank holds no excess liquidity, i.e., $L_1 = \lambda = 0.02$. They also show that under {Liquidity, Liquidation} the bank does hold excess liquidity.[8] The probability of a run is higher under {Liquidation, Liquidity} than under {Liquidity, Liquidation} because under the latter pecking order the negative effect of higher liquidity on the likelihood of a run (the first and third terms of (8)) dominates the positive effect (the second term of (8)). The lower occurrence of runs under {Liquidity, Liquidation} has the further effect of lowering the expected cost of suspension. Hence, under this pecking order the RA anticipates resolution (i.e., $\phi_2 < \phi_1$).

All in all, these two latter effects highlight that the economy under {Liquidation, Liquidity} is more fragile than under {Liquidity, Liquidation}. Nevertheless, the last column of Table 2 shows that the expected welfare is higher under the former than under the latter. In other words, the equilibrium ex-ante optimal pecking order is {Liquidation, Liquidity}. To understand why this happens despite higher fragility, we analyze the welfare decomposition in columns 4-6. First, under the pecking order {Liquidation, Liquidity}, the good bank after suspension receives a larger amount of productive assets, that generate higher proceeds and therefore higher welfare than under {Liquidity, Liquidation}

[7]Using return on assets yields qualitatively similar results. Source: MFI Interest Rate Statistics, European Central Bank.

[8]Notice that this result holds despite the fact that $r > 1/R$. This reflects the fact that the condition for excess liquidity in Proposition 2 is sufficient but not necessary.

(column 4). Second, when a run does not occur, under {Liquidity, Liquidation} the bank brings about more welfare from liquidity holdings than under {Liquidation, Liquidity}, because in this latter case, it holds zero excess liquidity (column 5). Third, the bank under {Liquidation, Liquidity} holds a substantially higher amount of productive assets than under {Liquidity, Liquidation}. Therefore, the welfare that the bank brings about from them is higher in the former case than in the latter (column 6).

To sum up, when choosing ex ante its asset pecking order rather than ex post, the bank further takes into account how this choice might affect its own portfolio and RA's resolution decisions. Put differently, there are two aspects driving the ex-ante choice of the pecking order, namely (i) the expected returns from productive investments when a run does not occur, and (ii) the type and amount of resources left to the good bank at resolution. Then, choosing ex ante the pecking order {Liquidation, Liquidity} allows more effective liquidity management for the bank if a run does not occur, and eventually for the good bank after suspension.

4.3 Comparative statics

In order to assess the generality of the previous results, we study their robustness to alternative parameter configurations in a comparative statics exercise. In particular, we let the return on the productive assets R vary in the interval $[2.02, 2.06]$ and the recovery rate r in the interval $[0.75, 0.85]$, to stay within the range of reasonable values.

The first row of Figure 2 shows that financial fragility, as represented by the threshold signals σ_1^* and σ_2^*, is decreasing in R for a given recovery rate r under both pecking orders, because a higher return on the productive asset lowers depositors' incentives to run. For given R instead, the effect of increasing the recovery rate r under the two pecking orders is of opposite sign. The reason is that two forces are at play. On the one hand, a higher r raises the bank's value at bankruptcy thereby delaying resolution (i.e., it increases the suspension points ϕ_1 and ϕ_2) and increasing depositors' incentives to run. On the other hand, a higher r also raises the amount of productive assets that are not liquidated, thus reducing depositors' incentives to run. The second channel dominates under {Liquidation, Liquidity}, while the first one dominates under {Liquidity, Liquidation}.

Suspension points ϕ_1 and ϕ_2 in the second row of Figure 2 are increasing in the recovery rate r because a higher r lowers the marginal benefits of an early suspension. For given r, the suspension point ϕ_1 is independent of R, as proven in Corollary 1. An increase in R has instead the effect of anticipating suspension under {Liquidity, Liquidation}, i.e., ϕ_2 declines. This is not due to a direct effect of R on the suspension point ϕ_2, as a

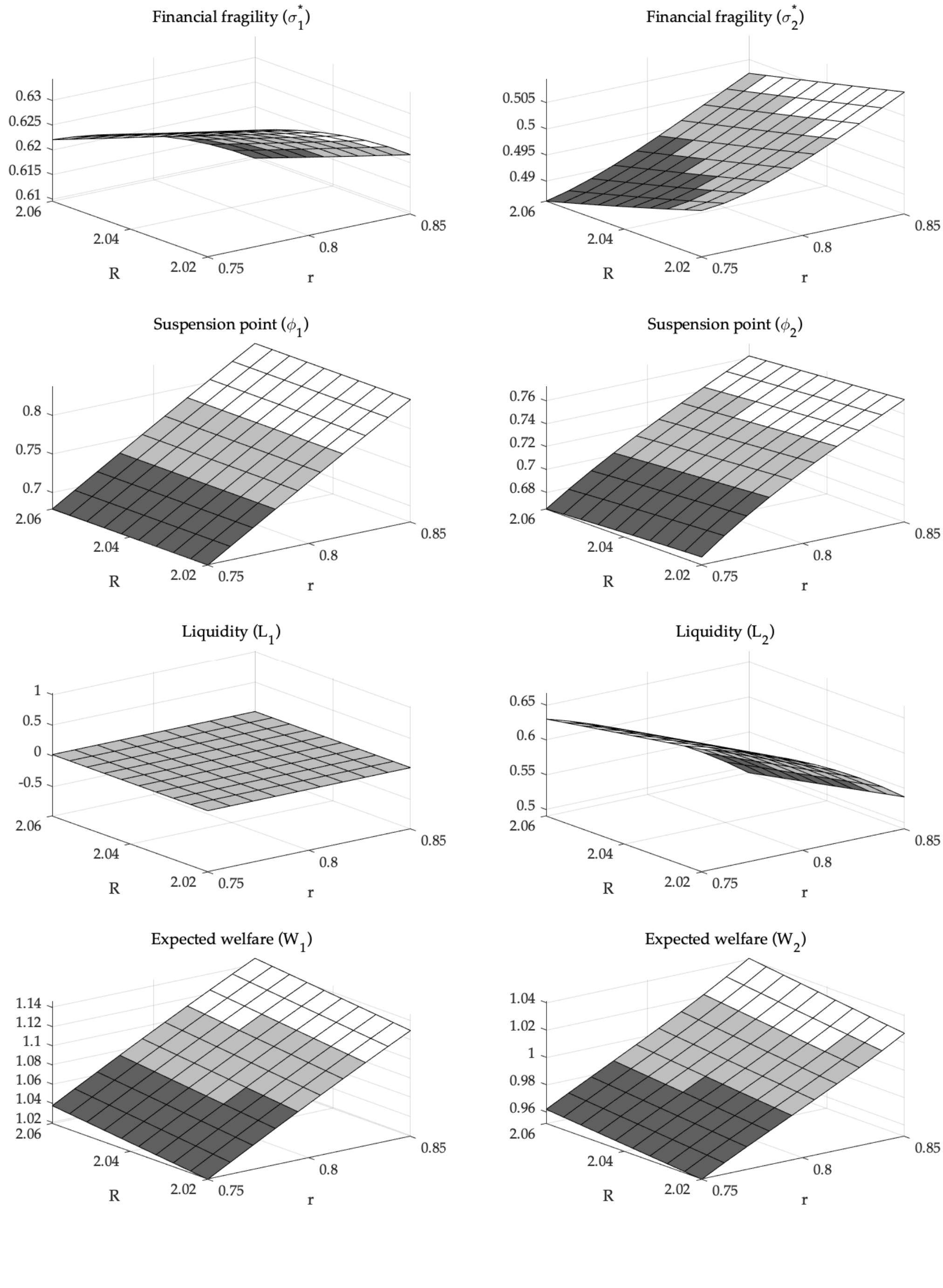

Figure 2: Comparative statics of the banking equilibrium with ex-ante optimal pecking order. The left column reports results under {Liquidation, Liquidity}. The right column reports results under {Liquidity, Liquidation}.

result, similar to Corollary 1 holds under {Liquidity, Liquidation} too (see equation (12)). However, a higher return on the productive asset has the effect of lowering the bank's liquidity holdings, and this in turn increases the RA's incentives to anticipate suspension.

The third row of Figure 2 reports the amount of liquidity chosen by the bank under the two pecking orders. As proved in Proposition 3, under {Liquidation, Liquidity} the bank holds no excess liquidity, irrespective of return on the productive asset and recovery rate. Under {Liquidity, Liquidation} instead, liquidity is decreasing in both parameters, as they both lower the opportunity cost of holding the productive asset.

Finally, the last row of Figure 2 shows that depositors' expected welfare under both pecking orders is increasing in the return on the productive asset and recovery rate, as higher values represent more productive investment technologies. Moreover, the result of the main numerical analysis is confirmed. The pecking order {Liquidation, Liquidity} is always preferred to {Liquidity, Liquidation} irrespective of the return on the productive asset and recovery rate.

4.4 Time inconsistency

The comparison between the previous two sections highlights a series of results. If the bank chooses the pecking order with which to use its assets ex post as a run unfolds, it finds it optimal to first deploy liquidity and then liquidate productive assets. When instead the bank chooses the pecking order ex ante, it recognizes that this decision impacts not only depositors' incentives to run, but also its own portfolio and RA's suspension decisions.

Note that the equilibrium allocation with the ex-post optimal pecking order is equivalent to the one under the pecking order {Liquidity, Liquidation} that we numerically evaluate in Section 4.2. Then, our analysis shows the existence of a plausible parameter space in which the expected welfare is higher when the bank chooses the pecking order ex ante rather than ex post.

Accordingly, we would expect the bank to choose the asset pecking order ex ante, and stick to this choice as a run unfolds. However, this behavior would be time inconsistent. To see that, assume that the bank commits to the pecking order {Liquidation, Liquidity} on date 0. Yet, if a run actually occurs on date 1, the pecking order {Liquidity, Liquidation} is the one that minimizes ex-post depositors' incentives to run, i.e., Proposition 1 still holds. In other words, the commitment to a pecking order on date 0 is not credible, and the bank first deploys liquidity and then liquidates productive assets as a run unfolds.

The last question remains of what is the mechanism underlying the time inconsistency. We argued in Section 4.2 that when choosing ex ante its asset pecking order, the bank further takes into account how it might affect its own portfolio and RA's resolution decisions. Put differently, the bank considers the expected returns from productive investments when a run does not occur, and the type and amount of resources left to the good bank at resolution. Then, to point out which of the two channels is at the core of the time inconsistency, in Appendix C we study an economy without any resolution procedure. In this case, if a run occurs the bank becomes insolvent and must liquidate all assets and equally share the proceeds among all depositors irrespective of the chosen pecking order.

In such a framework, {Liquidity, Liquidation} is still the pecking order of choice when the decision is taken ex post, as a similar result to Proposition 1 holds. Table 3 reports instead the numerical characterization of the equilibrium when the bank chooses the pecking order ex ante. A comparison of the welfare measures in Tables 3e and 3f shows that no time inconsistency arises, i.e., {Liquidity, Liquidation} is preferred to {Liquidation, Liquidity}. The main reason for this is that the former pecking order is associated with lower financial fragility (see Tables 3a and 3b). This also implies that the bank keeps holding excess liquidity in equilibrium, as the comparison between Tables 3c and 3d makes clear.

To sum up, the analysis conveys the message that what drives the time inconsistency in the choice of the pecking order is the bank's willingness to take into account the type and amount of resources left to the good bank if resolution occurs, rather than the expected returns if a run does not occur.

5 Concluding remarks

We have studied how regulation shapes the interaction between financial fragility and bank liquidity management and proposed a rationale for the complementarity between bank recovery and resolution planning. The novelty of our contribution lies in the analysis of the asset pecking order that banks follow to meet depositors' withdrawals during runs, and its interaction with resolution planning. Our results show that the RA's commitment to resolution planning makes it necessary to also impose a commitment on banks' liquidity management against financial fragility, as recovery planning does in the real world. Otherwise, such liquidity management would be subject to time inconsistency. In other words, it is the time inconsistency of banks' liquidity management that brings about the complementarity between recovery and resolution planning. All in all, recovery and resolution planning have the potential to lead to a more valuable resource allocation, by

Table 3: Equilibrium without resolution. The left column reports results under {Liquidation, Liquidity}. The right column reports results under {Liquidity, Liquidation}.

(a) Financial fragility (σ_1^*)

	$r = 0.75$	$r = 0.80$	$r = 0.85$
$R = 2.02$	0.8895	0.8109	0.7356
$R = 2.04$	0.8808	0.8030	0.7283
$R = 2.06$	0.8722	0.7952	0.7213

(b) Financial fragility (σ_2^*)

	$r = 0.75$	$r = 0.80$	$r = 0.85$
$R = 2.02$	0.6580	0.6743	0.6858
$R = 2.04$	0.6576	0.6729	0.6835
$R = 2.06$	0.6570	0.6714	0.6812

(c) Liquidity (L_1)

	$r = 0.75$	$r = 0.80$	$r = 0.85$
$R = 2.02$	0.0200	0.0200	0.0200
$R = 2.04$	0.0200	0.0200	0.0200
$R = 2.06$	0.0200	0.0200	0.0200

(d) Liquidity (L_2)

	$r = 0.75$	$r = 0.80$	$r = 0.85$
$R = 2.02$	0.5701	0.3927	0.1802
$R = 2.04$	0.5474	0.3732	0.1638
$R = 2.06$	0.5259	0.3545	0.1480

(e) Expected welfare (W_1)

	$r = 0.75$	$r = 0.80$	$r = 0.85$
$R = 2.02$	0.8804	0.9947	1.0870
$R = 2.04$	0.8915	1.0046	1.0960
$R = 2.06$	0.9025	1.0146	1.1051

(f) Expected welfare (W_2)

	$r = 0.75$	$r = 0.80$	$r = 0.85$
$R = 2.02$	1.0285	1.0548	1.0966
$R = 2.04$	1.0327	1.0605	1.1041
$R = 2.06$	1.0371	1.0664	1.1117

allowing more effective liquidity management by banks.[9]

Note that in our framework, there is no deposit insurance. As already mentioned, this can be justified by the increasing role of uninsured deposits on banks' balance sheets. Moreover, Allen et al. (2018) show that deposit insurance functioning as in the real world (i.e., as a guarantee of a fixed repayment in any possible state of the economy) would not completely rule out self-fulfilling runs. A similar argument holds regarding liquidity requirements. Only forcing banks to be "narrow" would make them immune from financial fragility. However, that would come at the cost of losing maturity transformation and possibly making banks redundant (Wallace, 1996). This is the reason why in the real

[9] In the real world banks, even if subject to recovery and resolution planning, hold significant amounts of liquidity. This is apparently at odds with the outcome of the model that, under recovery and resolution planning, banks hold no excess liquidity. Yet, in the real world banks are currently also subject to tight liquidity requirements, from which we abstract in the model.

world, liquidity requirements are milder and potentially leave some financial fragility unresolved. This means that the introduction of plausible deposit insurance schemes and liquidity requirements would alter neither the mechanism nor the conclusions of our argument.

Finally, it would be incorrect to draw normative implications on the efficiency of recovery and resolution planning, by comparing our results to an economy without such policy interventions. In fact, that would imply a comparison of the equilibrium outcomes of two models with different environments. Nevertheless, a still noteworthy policy implication can be drawn. Changing the architecture of recovery and resolution planning by limiting banks' independence and giving all powers to the RA, should not affect banks' liquidity management as long as banks are competitive and share with the RA the objective of maximizing depositors' welfare. The introduction of market power might change this argument, but remains an open question that we leave to future research.

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Appendices

A Proofs

Proof of Lemma 1. In the interval (λ, ϕ),

$$\frac{\partial v_1(\sigma, n)}{\partial n} = \sigma \frac{-\frac{R}{r}(1-n) + R\left(1 - L - \frac{n-\lambda}{r}\right)}{(1-n)^2} + \frac{L-\lambda}{(1-n)^2}. \tag{20}$$

This is negative if

$$(L-\lambda)(1-\sigma R) < \sigma R\left(\frac{\lambda r + (1-\lambda)}{r} - 1\right). \tag{21}$$

As $\sigma > \underline{\sigma} = 1/R$, then $\sigma R > 1$. Hence, the left side is negative, and the right side is positive.

In the interval (λ, n_2^*),

$$\frac{\partial v_2(\sigma, n)}{\partial n} = \sigma R \frac{1-L}{(1-n)^2} + \frac{L-1}{(1-n)^2}. \tag{22}$$

This is negative if $\sigma R < 1$ which is impossible because $\sigma > \underline{\sigma}$, where $\underline{\sigma} < 1$. In the interval $[n_2^*, \phi)$ instead,

$$\frac{\partial v_2(\sigma, n)}{\partial n} = \sigma \frac{-\frac{R}{r}(1-n) + R\left(1 - L - \frac{n-L}{r}\right)}{(1-n)^2} = \sigma R \frac{1-L}{(1-n)^2}\left(1 - \frac{1}{r}\right) < 0. \tag{23}$$

■

Proof of Proposition 1. A bank willing to maximize expected welfare by choosing the pecking order ex post should pick the one that minimizes depositors' incentives to run. Then, we want to prove that the pecking order {Liquidity, Liquidation} is preferred to {Liquidation, Liquidity} for any possible value of $n \in (\lambda, \phi)$. We know that for $n = \lambda$, $v_1(\sigma, \lambda) = v_2(\sigma, \lambda)$. Moreover, recall that by Lemma 1, $v_1(\sigma, n)$ is decreasing everywhere in $n \in (\lambda, \phi)$, and $v_2(\sigma, n)$ is first increasing in $n \in (\lambda, n_2^*)$ and then decreasing in $n \in (n_2^*, \phi)$. As a consequence, in the interval (λ, n_2^*), $v_2(\sigma, n) > v_1(\sigma, n)$.

In order to prove the proposition, we just need to show that the second arm of $v_2(\sigma, n)$ is higher than $v_1(\sigma, n)$ between n_2^* and ϕ, for given L. In fact:

$$\sigma \frac{R\left(1 - L - \frac{n-L}{r}\right)}{1-n} > \sigma \frac{R\left(1 - L - \frac{n-\lambda}{r}\right) + L - \lambda}{1-n} + (1-\sigma)\frac{L-\lambda}{1-n}. \tag{24}$$

Simplifying, we obtain $L \geq \lambda$ which is always true. ■

Proof of Proposition 2. We prove the proposition by contradiction. Assume that $L_2 =$

λ, so that $n_2^* = L_2 = \lambda$. Then:

$$\sigma_2^*(\lambda,\phi) = \frac{\phi-\lambda}{\int_\lambda^\phi \frac{R}{1-n}\left[1-\lambda-\frac{1}{r}(n-\lambda)\right]dn} > \frac{\phi-\lambda}{\int_\lambda^\phi \frac{R}{1-n}\left[1-\lambda-(n-\lambda)\right]dn} = \frac{1}{R}. \tag{25}$$

Moreover:

$$\left.\frac{\partial\sigma_2^*}{\partial L}\right|_{L=\lambda} = -\frac{\sigma_2^* R}{DEN_{\sigma_2^*}}\left(\frac{1}{r}-1\right)\int_\lambda^\phi \frac{1}{1-n}dn < 0. \tag{26}$$

So the third term of (13) is positive. The sum of the first two terms is equal to:

$$\frac{\sigma_2^{*2}}{2}R\left(\frac{1}{r}-1\right)+1-\sigma_2^*-R\frac{1-\sigma_2^{*2}}{2} = \frac{R}{2}\left[\frac{\sigma_2^{*2}}{r}-1\right]+1-\sigma_2^* \tag{27}$$

Under $r < 1/R^2$ this expression is positive. Hence $L_2 = 1 > \lambda$, which is a contradiction.

■

Proof of Lemma 2. Taking the derivative of (14) with respect to ϕ_1 and L_1 yields:

$$\frac{\partial\sigma_1^*}{\partial\phi_1} = \frac{1}{DEN_{\sigma_1^*}}\left[1-\frac{L_1-\lambda}{1-\phi_1}-\sigma_1^* R\frac{1-L_1-\frac{\phi_1-\lambda}{r}}{1-\phi_1}\right], \tag{28}$$

$$\frac{\partial\sigma_1^*}{\partial L_1} = \frac{1}{DEN_{\sigma_1^*}}(\sigma_1^* R-1)\int_\lambda^{\phi_1}\frac{1}{1-n}dn. \tag{29}$$

The first expression is positive as $v_1(\sigma_1^*,\phi_1) < 0$. The second expression is also positive as by construction $\sigma_1^* > \underline{\sigma} = 1/R$. ■

Proof of Corollary 1. Using (14), (16) and (28) into (17), we obtain:

$$-\left[\phi_1-\lambda-\int_\lambda^{\phi_1}\frac{L_1-\lambda}{1-n}dn\right]\psi'(\phi_1) = \left[\frac{\phi_1-\lambda-\int_\lambda^{\phi_1}\frac{L_1-\lambda}{1-n}dn}{\int_\lambda^{\phi_1}\frac{\left(1-L_1-\frac{n-\lambda}{r}\right)}{1-n}dn}\frac{1}{2r}-1\right]\times$$

$$\times\left[\phi_1-\lambda-\int_{\lambda}^{\phi_1}\frac{L_1-\lambda}{1-n}dn\right]+\left[1-\frac{L_1-\lambda}{1-\phi_1}-\frac{\phi_1-\lambda-\int_{\lambda}^{\phi_1}\frac{L_1-\lambda}{1-n}dn}{\int_{\lambda}^{\phi_1}\frac{\left(1-L_1-\frac{n-\lambda}{r}\right)}{1-n}dn}\frac{1-L_1-\frac{\phi_1-\lambda}{r}}{1-\phi_1}\right]\times$$
$$\left[\frac{\phi_1-\lambda-\int_{\lambda}^{\phi_1}\frac{L_1-\lambda}{1-n}dn}{\int_{\lambda}^{\phi_1}\frac{\left(1-L_1-\frac{n-\lambda}{r}\right)}{1-n}dn}(1-L_1)-\phi_1-\frac{\phi_1-\lambda-\int_{\lambda}^{\phi_1}\frac{L_1-\lambda}{1-n}dn}{\int_{\lambda}^{\phi_1}\frac{\left(1-L_1-\frac{n-\lambda}{r}\right)}{1-n}dn}\left(1-L_1-\frac{\phi_1-\lambda}{r}\right)+\lambda+\psi(\phi_1)\right]. \tag{30}$$

This expression implicitly characterizes the equilibrium suspension point ϕ_1 as a function of L_1. As the latter is equal to λ in equilibrium, and R does not appear in (30), it must be the case that ϕ_1 is independent of R. ■

B Equilibrium with perfect information

In this appendix, we characterize a banking equilibrium in which the bank is perfectly informed about depositors' types, i.e., it can observe and verify the realizations of the idiosyncratic shocks hitting the depositors (but not the realization of the aggregate state) and maximizes their expected welfare subject to budget constraints. This is a reasonable benchmark to the main analysis, as its equilibrium outcome is equivalent to that of an economy with imperfect information in which banks cannot observe depositors' types and must induce truth-telling, but depositors do not observe noisy signals about the aggregate state. Formally, the bank solves the following:

$$\max_{c_2(Z),L,D}\lambda+(1-\lambda)\mathbb{E}[c_2(Z)], \tag{31}$$

subject to

$$L+rD\geq\lambda, \tag{32}$$

$$0 \leq D \leq 1 - L, \tag{33}$$

$$L \geq 0, \tag{34}$$

$$(1-\lambda)c_2(Z) = Z(1 - L - D) + L + rD - \lambda, \tag{35}$$

where the last constraint has to hold for any $Z \in \{0, R\}$. On date 0, the bank collects all endowments and invests them in an amount L of liquidity and $1 - L$ of productive assets. On date 1, the liquidity constraint (32) states that the amount of liquid assets, given by the sum of liquidity and the resources generated by liquidating an amount D of productive assets, must be sufficient to pay early consumption $c = 1$ to λ early consumers. Any resource $L + rD - \lambda$ left constitutes excess liquidity and is rolled over to date 2. The excess liquidity, together with the return from the remaining productive assets, pays for late consumption according to (35) for any realization of the aggregate productivity shock $Z \in \{0, R\}$. The bank cannot liquidate a negative amount of assets and cannot liquidate more assets than are available on its balance sheet.

Substituting the budget constraints into the objective function yields the following banking problem:

$$\max_{L,D} \; \lambda + (1-\lambda) \int_0^1 \left[p \frac{R(1-L-D) + L + rD - \lambda}{1-\lambda} + (1-p) \frac{L + rD - \lambda}{1-\lambda} \right] dp, \tag{36}$$

subject to $L + rD \geq \lambda$, $L \geq 0$ and $0 \leq D \leq 1 - L$. The following result holds:

Lemma 3. *In the banking equilibrium with perfect information, there is no liquidation of the productive asset and no excess liquidity, i.e.,* $D = 0$ *and* $L = \lambda$.

Proof. Simplify the objective function as follows:

$$\max_{L,D} \; L + rD + \int_0^1 pR(1 - L - D), \tag{37}$$

subject to $L + rD \geq \lambda$, $L \geq 0$, $D \leq 1 - L$ and $D \geq 0$. Assign Lagrange multipliers ξ, μ, η and ζ, respectively. The first-order conditions are

$$L: \qquad 1 - \mathbb{E}[p]R + \xi + \mu - \eta = 0, \tag{38}$$

$$D: \qquad r - \mathbb{E}[p]R + r\xi - \eta + \zeta = 0. \tag{39}$$

Substituting the first into the second, we obtain

$$-\mathbb{E}[p](1 - r)R - r\mu - (1 - r)\eta + \zeta = 0. \tag{40}$$

Hence, it must be that $\zeta > 0$ and $D = 0$ by complementary slackness. According to the liquidity constraint (32), this means that $L \geq \lambda > 0$. Thus, $\mu = 0$. Then, from (38) it follows that

$$\xi = \mathbb{E}[p]R - 1 + \eta, \tag{41}$$

which is strictly positive, as $\mathbb{E}[p]R > 1$. Therefore, $L = \lambda < 1$ and $\eta = 0$. ■

The above lemma states that liquidating productive assets to create liquidity on date 1 is never part of an equilibrium in the case of perfect information, because the recovery rate $r < 1$ implies that liquidation is too costly. Consequently, banks only use liquidity to serve early consumers. Moreover, the advantage of holding excess liquidity, in terms of consumption in the bad state of the world when the productive asset yields zero, is dominated by its cost in terms of forgone return in the good state of the world. Hence, the bank in equilibrium holds just enough liquidity to cover the total early consumption λ.

C Equilibrium without resolution

The aim of this appendix is to characterize the equilibrium of an economy in which the bank still chooses the pecking order with which to employ assets during runs (either ex ante or ex post) but without the intervention of a resolution procedure. In this case, if a run occurs the bank becomes insolvent and must liquidate all assets and equally share the proceeds among all depositors, irrespective of the pecking order. Mimicking the structure of the main text, we first solve for the equilibrium with ex-post choice of the pecking order, and then for the equilibrium with ex-ante choice.

C.1 Equilibrium with ex-post optimal pecking order

We solve the model by backward induction. First, for a given asset pecking order and bank liquidity, we study the depositors' withdrawal decisions. Then, we characterize the equilibrium asset pecking order, and finally, the choice of the bank asset portfolio.

All the considerations of the main text regarding the upper and lower dominance region still hold. To characterize depositors' withdrawal behavior, we first study the utility advantage of waiting over running under both pecking orders, for a given fraction n of depositors who withdraw funds on date 1. Table 4 reports payoffs under the two pecking orders. Under the pecking order {Liquidation, Liquidity}, $n_1^* = \lambda + r(1-L)$ and $n^{**} = L + r(1-L)$ are the maximum fractions of depositors that a bank can serve on date 1, and either liquidating the whole amount of productive assets in the portfolio (up to n_1^*) or also using liquidity (up to n^{**}). If the fraction of depositors who withdraw funds on date 1 is in the interval $[n^{**}, 1]$, the bank becomes insolvent, as it does not hold sufficient resources to repay all depositors on date 1. In this case, the bank is forced to liquidate all productive assets and close down. Therefore, a late consumer who waits until date 2 receives zero. The total liquidation value of the bank's assets (equal to $L + r(1-L)$) is split pro-rata among the n depositors withdrawing funds on date 1.

Table 4: Depositors' ex-post payoffs in the economy without resolution.

(a) Pecking order {Liquidation, Liquidity}

Date	$n \in [\lambda, n_1^*)$	$n \in [n_1^*, n^{**})$	$n \in [n^{**}, 1]$
$t = 1$	1	1	$\frac{r(1-L)+L}{n}$
$t = 2$	$\frac{Z\left(1-L-\frac{n-\lambda}{r}\right)+L-\lambda}{1-n} \quad \forall Z \in \{0, R\}$	$\frac{r(1-L)+L-n}{1-n}$	0

(b) Pecking order {Liquidity, Liquidation}

Date	$n \in [\lambda, n_2^*)$	$n \in [n_2^*, n^{**})$	$n \in [n^{**}, 1]$
$t = 1$	1	1	$\frac{r(1-L)+L}{n}$
$t = 2$	$\frac{Z(1-L)+L-n}{1-n} \quad \forall Z \in \{0, R\}$	$\frac{Z\left(1-L-\frac{n-L}{r}\right)}{1-n} \quad \forall Z \in \{0, R\}$	0

Accordingly, consumption at insolvency is $c^B(n) = (L + r(1 - L))/n$. Under the pecking order {Liquidity, Liquidation} instead, $n_2^* = L_2$ and $n^{**} = L + r(1 - L)$ are the maximum fractions of depositors that a bank can serve on date 1 by deploying liquidity (up to n_2^*), and by also liquidating the whole amount of productive assets in its portfolio (up to n^{**}).

Given the described payoff structure, under the pecking order {Liquidation, Liquidity} the utility advantage of waiting over running is

$$v_1(\sigma, n) = \begin{cases} \sigma \frac{R\left(1-L-\frac{n-\lambda}{r}\right)+L-\lambda}{1-n} + (1-\sigma)\frac{L-\lambda}{1-n} - 1 & \text{if } \lambda \leq n < n_1^*, \\ \sigma \frac{r(1-L)+L-n}{1-n} + (1-\sigma)\frac{r(1-L)+L-n}{1-n} - 1 & \text{if } n_1^* \leq n < n^{**}, \\ -\frac{r(1-L)+L}{n} & \text{if } n^{**} \leq n \leq 1, \end{cases} \tag{42}$$

while under the pecking order {Liquidity, Liquidation} it is

$$v_2(\sigma, n) = \begin{cases} \sigma \frac{R(1-L)+L-n}{1-n} + (1-\sigma)\frac{L-n}{1-n} - 1 & \text{if } \lambda \leq n < n_2^*, \\ \sigma \frac{R(1-L-D)}{1-n} - 1 = \sigma \frac{R\left(1-L-\frac{n-L}{r}\right)}{1-n} - 1 & \text{if } n_2^* \leq n < n^{**}, \\ -\frac{r(1-L)+L}{n} & \text{if } n^{**} \leq n \leq 1. \end{cases} \tag{43}$$

The strategic complementarities affecting a late consumer's decision to run depend on how the advantage of waiting over running varies with the fraction of depositors n withdrawing funds on date 1. Under both pecking orders, in the interval $[n^{**}, 1]$ the advantage of waiting over running is increasing in n, as after insolvency, equal service prescribes the total resources to be shared pro-rata with all depositors. The following lemma characterizes the strategic complementarities in the other intervals under the two pecking orders.

Lemma 4. *The function $v_1(\sigma, n)$ is decreasing in $n \in (\lambda, n^{**})$. The function $v_2(\sigma, n)$ is increasing in $n \in (\lambda, n_2^*)$ and decreasing in $[n_2^*, n^{**})$.*

Proof. In the interval (λ, n_1^*),

$$\frac{\partial v_1(\sigma, n)}{\partial n} = \sigma \frac{-\frac{R}{r}(1-n) + R\left(1 - L - \frac{n-\lambda}{r}\right) + R(L-\lambda) - R(L-\lambda)}{(1-n)^2} + \frac{L-\lambda}{(1-n)^2}. \tag{44}$$

This is negative if

$$(L-\lambda)(1-\sigma R) < \sigma R\left(\frac{\lambda r + (1-\lambda)}{r} - 1\right). \tag{45}$$

As $\sigma > \underline{\sigma} = 1/R$, then $\sigma R > 1$. Hence, the left-hand side is negative, and the right-hand side is positive. In the interval $[n_1^*, n^{**})$, $v_1(\sigma, n)$ is decreasing, as $1 > L + r(1-L)$. In the interval (λ, n_2^*),

$$\frac{\partial v_2(\sigma, n)}{\partial n} = \sigma R \frac{1-L}{(1-n)^2} + \frac{L-1}{(1-n)^2}. \tag{46}$$

This is negative if

$$1 - L > \sigma R(1 - L), \tag{47}$$

which is impossible because $\sigma > \underline{\sigma}$. In the interval $[n_2^*, n^{**})$ instead,

$$\frac{\partial v_2(\sigma, n)}{\partial n} = \sigma \frac{-\frac{R}{r}(1-n) + R\left(1 - L - \frac{n-L}{r}\right)}{(1-n)^2} = \sigma R \frac{1-L}{(1-n)^2}\left(1 - \frac{1}{r}\right) < 0. \tag{48}$$

■

Figure 3 shows that under both pecking orders the advantage of waiting over running crosses zero only once, thus guaranteeing the existence and uniqueness of the equilibrium (Goldstein and Pauzner, 2005). A result similar to Proposition 1 holds:

Proposition 4. *If $n \in (\lambda, n^{**})$, the pecking order {Liquidity, Liquidation} is always preferred to {Liquidation, Liquidity}.*

As before, a bank willing to maximize expected welfare by choosing the pecking order ex post (i.e., after its portfolio and RA's suspension decisions) should pick the one that minimizes depositors' incentives to run. Then, Proposition 4 implies that the equilibrium run threshold with ex-post optimal pecking order is the one that solves $\mathbb{E}[v_2(\sigma, n)|\sigma_2^*] = 0$, i.e.,

$$\sigma^*(L) = \sigma_2^*(L) = \frac{n^{**} - \lambda - \int_\lambda^{n_2^*} \frac{L-n}{1-n} dn + \int_{n^{**}}^1 \frac{r(1-L)+L}{n} dn}{\int_\lambda^{n_2^*} \frac{R(1-L)}{1-n} dn + \int_{n_2^*}^{n^{**}} \frac{R\left(1-L-\frac{n-L}{r}\right)}{1-n} dn}. \tag{49}$$

By backward induction, we solve for the equilibrium bank asset portfolio in the following:

$$\max_L \int_0^{\sigma^*(L)} \left[L + r(1-L)\right] dp + \int_{\sigma^*(L)}^1 \left[\lambda + (1-\lambda)\left[p\frac{R(1-L) + L - \lambda}{1-\lambda} + (1-p)\frac{L-\lambda}{1-\lambda}\right]\right] dp, \tag{50}$$

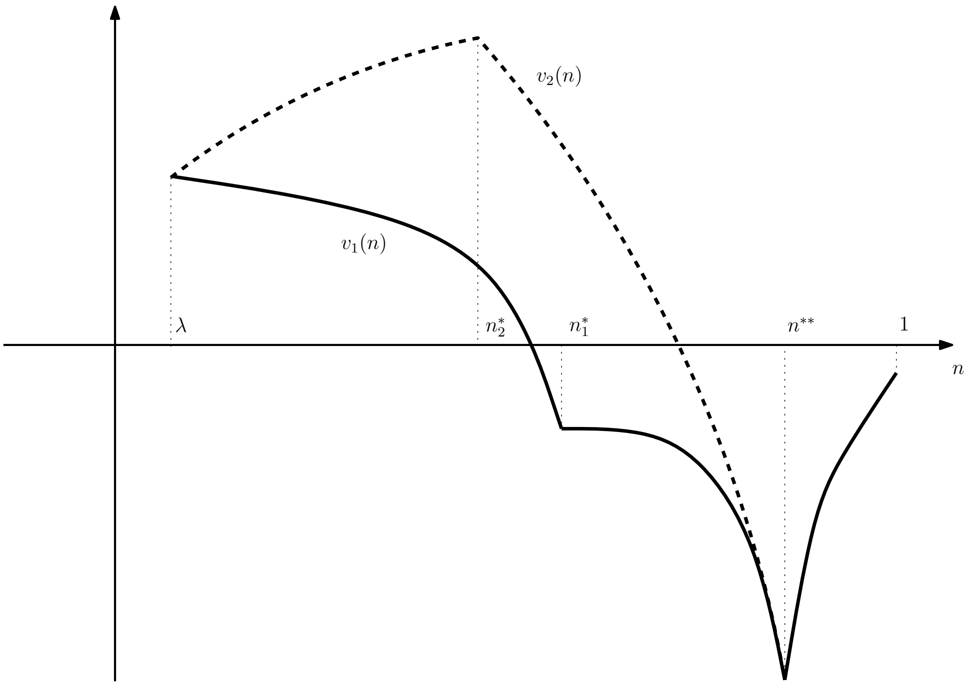

Figure 3: The advantage of waiting over running, as a function of the fraction of depositors running, under the pecking orders {Liquidation, Liquidity} (solid line) and {Liquidity, Liquidation} (dashed line).

subject to $\lambda \leq L \leq 1$. Define the difference between the utility in the no-run case and the utility in the case of a run as

$$\Delta U = L + \sigma^*(L)R(1-L) - L - r(1-L) = (\sigma^*(L)R - r)(1-L). \tag{51}$$

Then, the first-order condition of the optimization problem with respect to L is

$$\sigma^*(L)(1-r) + \int_{\sigma^*(L)}^{1} \big[-pR+1\big]dp - \frac{\partial \sigma^*}{\partial L}\Delta U + \xi - \chi = 0, \tag{52}$$

where ξ and χ are the respective Lagrange multipliers on the two constraints.

C.2 Equilibrium with ex-ante optimal pecking order

As in the main text, to characterize the ex-ante optimal pecking order, we first need to characterize the run threshold $\sigma_1^*(L)$ from $\mathbb{E}[v_1(\sigma, n)|\sigma_1^*] = 0$:

$$\sigma_1^*(L) = \frac{n^{**} - \lambda - \int_\lambda^{n_1^*} \frac{L-\lambda}{1-n} dn - \int_{n_1^*}^{n^{**}} \frac{L+r(1-L)+L-n}{1-n} dn + \int_{n^{**}}^{1} \frac{r(1-L)+L}{n} dn}{\int_\lambda^{n_1^*} \frac{R\left(1 - L - \frac{n-\lambda}{r}\right)}{1-n} dn} \tag{53}$$

The banking problem is the same as in (50) with a different run threshold and yields a similar first-order condition as in (52) for the choice of liquidity. The numerical characterization of the equilibrium with no resolution is reported in Table 3 in the main text.